

# Static and Dynamic Underinvestment: An Experimental Investigation\*

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## Abstract

In this paper, we analyze a stylized version of an environment with public goods, dynamic linkages, and legislative bargaining. Our theoretical framework studies the provision of a durable public good as a modified two-period version of Battaglini et al. (2012). The experimental design allows us to disentangle inefficiencies that would result in a one-shot world (static inefficiencies) from additional inefficiencies that emerge in an environment in which decisions in the present affect future periods (dynamic inefficiencies). We solve the first-best solution and compare it to the symmetric stationary subgame-perfect equilibrium of a legislative bargaining game. The experimental results indicate that subjects do react to dynamic linkages, and, as such, there is evidence of both static and dynamic inefficiencies. The quantitative predictions of the bargaining model with respect to the share of dynamic inefficiencies are closest to the data when dynamic linkages are high. To the extent that behavior is different from the model's predictions, a systematic pattern emerges - namely, the use of *strategic cooperation*, whereby groups increase the efficiency of current proposals by selectively punishing, in future proposals, individuals who propose highly inefficient allocations.

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# 1 Introduction

Many important public goods are supplied by the government and, thus, are determined via a legislative process. Furthermore, most of these public goods are long-lived and cannot be appropriately considered in the context of a one-shot decision. Rather, over time, the legislature must repeatedly determine the allocation of resources to public goods, and prior investments in such goods have impacts beyond the moment when the investment is made. Recent developments in economic theory integrate these factors into models of public good provision with dynamic linkages. Indeed, papers in political economy, such as Battaglini and Coate (2008), recognize the importance of dynamic linkages and provide an analysis of these type of situations. Once the setting is a dynamic one, multiple channels generate inefficiencies, or differences between the equilibrium level of public goods and the level that a central planner would select. In particular, agents wish to free-ride not only with respect to other agents' contributions in the current period, but also with respect to their future contributions.

In this paper, we design a stylized version of an environment with public goods, dynamic linkages, and legislative bargaining. The goal is to simplify the environment while maintaining some of the key features present in models such as Battaglini and Coate (2008). More precisely, our theoretical framework studies the provision of a durable public good as a modified version of Battaglini et al. (2012). We develop an experimental design that allows us to disentangle inefficiencies that would result in a one-shot world (static inefficiencies) from extra inefficiencies that emerge in an environment in which decisions in the present affect the future (dynamic inefficiencies). Note that such a question is particularly relevant given the frequent observation that free-riding is much less severe in public good experiments than the theory suggests. Hence, one may wonder whether people even take into consideration the more subtle issue of *dynamic* free riding. The setting we propose to investigate this question is simple. In each of two time periods, a committee decides on the allocation of a fixed budget over a public good and private consumption for each member. The division is determined by majority rule, using the multilateral bargaining procedure of Baron and Ferejohn (1989).<sup>1</sup> The dynamic link is provided by the public good, as a portion  $\delta \in (0, 1)$  of the first-period investment survives and is available in period 2. In other words, the level of the public good in period 2 equals the portion that survived from period 1, plus the period 2 investment. We solve for efficiency and also characterize the bargaining equilibrium, a symmetric stationary subgame perfect equilibrium, which is the most common concept used in applied work.

In the bargaining equilibrium, investment is distorted away from the first best. To see why, consider, first, the case in which no portion of the public good survives ( $\delta = 0$ ). When the planner decides how to allocate the budget, she considers the benefit that an additional unit of investment has for *all* committee members. With bargaining being settled by majority rule, however, the equilibrium results from computing the investment benefits to a *minimum winning coalition* (MWC). The consequence is underinvestment or static inefficiencies.

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<sup>1</sup>One member chosen at random submits an allocation proposal that all committee members then vote on. If the proposal does not achieve a simple majority of votes, it is rejected, and the process repeats itself.

When the public good is durable ( $\delta \in (0, 1)$ ), there is an obvious incentive for higher investment in period 1, but also new sources of underinvestment. A suboptimal period 1 choice will affect future choices. In the bargaining equilibrium, the committee will start in period 2 with a lower level of the public good, constraining the set of options for that period with respect to the efficient solution. The planner in a dynamic setting considers the effect that current decisions will have on all committee members in the present *and* in the future. But, again, in the bargaining equilibrium, period 1 decisions are concerned only with the present and the future of a subset of members - namely, those in the MWC.

We will use the term *dynamic inefficiencies* to refer to any underinvestment that results on top of static inefficiencies, with *static inefficiencies* being the ones that emerge when public goods do not affect payoffs for more than a single period. In the bargaining equilibrium, dynamic inefficiencies can represent a very large portion of total inefficiencies. For example, in our parametrization, if only 80% of period 1 investment survives in period 2, dynamic inefficiencies account for approximately three quarters of the total inefficiencies. Our theoretical environment provides a very conservative measure of dynamic underinvestment. First, we use a two-period model, as it is the simplest environment in which dynamic effects arise, but the inefficiency gap increases with the time horizon. Second, in our experiments, committees have three members, but dynamic inefficiencies increase with committee size. In other words, findings in line with dynamic inefficiency predictions in our setting suggest an even greater role for the practical relevance of dynamic effects in general.

Our experimental design allows us to disentangle the static from the dynamic component. The control treatment sets  $\delta = 0$  and provides us with a measure of static underinvestment. We also study the cases  $\delta \in \{0.2, 0.8\}$  in the laboratory, so that we can compute dynamic underinvestment in two treatments. Moreover, our treatments allow us to study the comparative statics of  $\delta$ . As the value of  $\delta$  increases, dynamic inefficiencies as a share of total inefficiencies also increase in the bargaining equilibrium prediction.

Our main finding is that dynamic inefficiencies can be large and increase with the dynamic link ( $\delta$ ). Specifically, when  $\delta = 0.8$ , dynamic inefficiencies as a share of total inefficiencies are at 75%, quite close to the theoretical prediction of 72%. Such inefficiencies decrease for lower values of the dynamic link. Several results contribute to these findings. First, subjects invest similar amounts in both periods of the static game ( $\delta = 0$ ), while public good investments in period 1 are significantly higher than those observed in period 2 in both dynamic games ( $\delta > 0$ ), reflecting the fundamental difference between dynamic and static bargaining environments. Second, period 1 public investments increase monotonically with the survival rate of the public good  $\delta$ . Third, we distinguish between two types of dynamic inefficiencies: the durability effect, which arises when the dynamic linkage between periods is strong ( $\delta = 0.8$ ), and the crowding-out effect, which arises when dynamic linkage between periods is relatively weak ( $\delta = 0.2$ ). Our data indicate that dynamic inefficiencies due to the durability effect are significant and large in magnitude, while dynamic inefficiencies due to the crowding-out effect are observed only if one focuses on the subset of proposals that involve MWCs.

Analyzing the individual behavior of subjects, we detect significant heterogeneity in proposals

types. First, in dynamic bargaining environments, there is a sizable presence of period 1 non-MWC proposals that benefit all members equally, with investment levels close to efficient ones. The presence of such proposals reduces inefficiencies and, in particular, explains why we observe dynamic inefficiencies only for MWC proposals when  $\delta = 0.2$ . Second, as  $\delta$  increases, there is more heterogeneity in period 1 investment.<sup>2</sup> A prominent finding in our dynamic treatments is the presence of strategic cooperation. Most subjects who make proposals with period 1 investment close to efficiency propose an MWC in period 2 - i.e., they are not unconditional altruists. On average, those subjects use the period 2 proposal to punish the period 1 proposers who did not select high investment and to reward those who did by strategically including them in period 2's MWC.

Our work is related to previous work on public good provision in static and dynamic settings. With respect to the former, our baseline treatment provides a setup in which the main theoretical prediction and the efficient solution involve interior investment levels. These features contrast with the usual framework used to study public goods: the linear public good game (or voluntary contribution mechanism – VCM). In that model, the dominant strategy and the efficient outcome are at the boundary of the action space, with an equilibrium prediction of no investment and an efficient outcome involving full provision. In such a setup, experiments show that investment in the public good remains positive even when participants have experience (see Ledyard (1995) and Vesterlund (2013)). Recent studies of public goods games modify the original VCM to have an interior solution in dominant strategies (see, for instance, Menietti et al. (2014)). Our work adds to this literature by also providing a static framework with interior solutions in which to study public good provision. Despite the equilibrium of our game not being in dominant strategies, contributions to the public good in the static treatment are much closer to equilibrium levels than in typical VCM experiments. Similarly, studies such as Menietti et al. (2014) report results close to equilibrium. The fact that both ours and other studies find congruent results in this regard highlight the importance of the specific details of the game in earlier results.

Our paper also relates to the literature on public good provision in committees. Two recent papers investigate public good provision in static environments. Fréchette et al. (2012) implement in the laboratory the Volden and Wiseman (2006) model of static public good provision with multilateral bargaining. The authors find that public good provision is closely related to the relative weight that subjects put on private versus public goods, consistent with the predictions of Markov perfect equilibrium. Christiansen et al. (2014) conduct an experimental study of the Jackson and Moselle (2002) model, in which players bargain over a single policy dimension and vary as to whether or not the proposer has access to a budget (pork) that she can privately allocate among committee members. The authors find that the introduction of private goods increases total welfare and shifts the location of the public policy issue from the median towards the location preferred by the most extreme member, who cares the most about the public policy issue.

Several papers study public good provision in dynamic settings. Contributions in this area include Herr et al. (1997), Noussair and Matheny (2000), Lei and Noussair (2002), Battaglini and

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<sup>2</sup>Also, in dynamic environments, there is no evidence of large proposer power in private allocations.

Palfrey (2012), Battaglini et al. (2012, 2013, 2014a), Saijo et al. (2014) and Vespa (2015).

Of these, the most closely related to our paper is Battaglini et al. (2012), which analyzes, both theoretically and experimentally, an infinite horizon model of the accumulation of a durable public good under different voting rules, using a different multilateral bargaining mechanism from ours. Its main theoretical result, which finds strong support in the experimental data, is that a higher majority requirement for passing proposals leads to more efficient public good investment. Aggregate levels of public investment are close to the predictions of the solution concept most commonly used in the literature (Markov perfect equilibrium), and behavior reflects non-myopic decision making. The motivation for the present paper is different: to study a simple two-period environment in which it is possible to disentangle the static from the dynamic forces behind underinvestment in a transparent way. This simple two-period framework preserves the main trade-offs that are present in the dynamic setting with durable public goods and allows for the characterization of more complex history-dependent strategies that have a substantial effect on the intertemporal pattern of investment.<sup>3</sup> While our study and Battaglini et al. (2012) were designed with very different objectives in mind, there are some interesting similarities. Both studies find that inefficiencies are pervasive, with levels of public good investment significantly below the optimal level. The types of proposals observed are also comparable, with most proposals involving side payments to minimum (or nearly minimum) winning coalitions.

The plan of the paper is as follows: Section 2 outlines the model that serves as our benchmark. Sections 3, 4 and 5 give the experimental design, the aggregate results, and the individual data analysis, respectively. A brief discussion of our results is reported in Section 6.

## 2 Theoretical Framework

As mentioned in the introduction, the model is meant to simplify dynamic models of public good provision while retaining the key strategic tensions of such an environment. The game is a two-period model ( $t = 1, 2$ ) of multilateral bargaining with  $n$  (odd) committee members indexed by  $i$ , each representing one district. There is no discounting between periods. In period  $t$ , the committee decides how to allocate a fixed budget  $B_t$  between pork to each member, denoted by  $x_{it}$ , and investment in a durable public good,  $I_t$ . Furthermore, consumption and investment must be non-negative in both periods, and there is no borrowing or lending. That is,  $x_{it} \geq 0$ ,  $I_t \geq 0$ ,  $\sum_{i=1}^n x_{it} + I_t \leq B_t$ ,  $t = 1, 2$ . Investment accumulates over time, and the resulting stock represents the level of the public good,  $g_t$ .

The utility of member  $i$  in period  $t$  is given by

$$U_{it}(x_{it}, g_t) = x_{it} + u(g_t),$$

where  $u(g_t)$  represents utility from the public good investment. We assume that  $u$  is twice con-

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<sup>3</sup>As shown in Section 5, considering history-dependent strategies is also useful for explaining the heterogeneity observed in the experimental data.

tinuously differentiable and has standard properties:  $u'(g_t) > 0$ ,  $u''(g_t) < 0$  for all  $g_t > 0$  and  $\lim_{g_t \rightarrow 0} u'(g_t) = \infty$ . The committee starts with zero stock of public good ( $g_0 = 0$ ), and in period 1, the level of the public good is equal to that period's investment ( $g_1 = I_1$ ). A portion  $\delta \in [0, 1]$  of the first period's investment survives and is still available in period 2, so the depreciation rate is  $d = 1 - \delta$ . The stock of the public good in period 2 is given by  $g_2 = \delta g_1 + I_2 = \delta I_1 + I_2$ . If the public good does not fully depreciate ( $\delta > 0$ ) between periods, the problem is dynamic.

In the remainder of this section, we characterize public good investments that arise from the bargaining process and compare them with efficient levels implemented by the social planner. Here, we present the main trade-offs of the model and refer the reader to Appendix A for complete proofs. Our discussion focuses on understanding the sources and determinants of the inefficiencies in public good provision due to the dynamic nature of the bargaining process.

## 2.1 Efficient Solution

The planner chooses investment levels and private allocations for each member so as to maximize society's welfare (aggregate utility of agents), subject to budget constraints and the rule that governs the accumulation of the stock of public good. Since agents' utilities depend linearly on private allocations, the planner's solution pins down the efficient level of investment in each period ( $I_1^{P*}, I_2^{P*}$ ) but is silent on how the remaining funds are distributed between agents in private shares.

$$\begin{aligned} & \max_{(\{x_{ij}, I_j^P\}_{j=1,2}^{i=1,\dots,n})} \left[ \sum_{i=1}^n x_{i1} + n \cdot u(g_1) + \sum_{i=1}^n x_{i2} + n \cdot u(g_2) \right] \\ \text{s.t. } & \sum_{i=1}^n x_{i1} + I_1^P \leq B_1 \text{ and } \sum_{i=1}^n x_{i2} + I_2^P \leq B_2 \\ & I_1^P \geq 0, I_2^P \geq 0, x_{ij} \geq 0 \forall i, j, \\ & \text{where } g_1 = I_1^P \text{ and } g_2 = \delta g_1 + I_2^P. \end{aligned}$$

Budget constraints are always binding, but the planner's solution depends on whether other constraints are binding or not. If no other constraints are binding, then there is an interior solution, ( $I_1^{P*}, I_2^{P*}$ ), characterized by two first-order conditions that capture a familiar trade-off:

$$\text{period 2: } \quad n \cdot u'(\delta I_1^{P*} + I_2^{P*}) = 1. \quad (1a)$$

$$\text{period 1: } \quad n \cdot \left[ u'(I_1^{P*}) + \delta u'(\delta I_1^{P*} + I_2^{P*}) \right] = 1. \quad (1b)$$

In each period, the interior level of public good provision equates the social marginal value of an additional unit of investment and its social marginal cost, which equals 1 in both periods. The marginal benefit of an extra unit of public good in period 2 is simply  $n \cdot u'(\delta I_1^{P*} + I_2^{P*})$ , as it benefits all  $n$  members of the committee equally. In period 1, however, there is an additional term, which

represents the effect of the public investment in period 1 that partially survives until period 2.<sup>4</sup>

If the solution is not interior, then which constraint is binding depends on parameters  $(B_1, B_2, \delta)$ . When available budgets  $(B_1, B_2)$  are sufficiently small, the planner allocates all available funds to public good provision. If this is not the case, then, depending on the depreciation rate, the planner might choose to allocate a portion of period 2's budget to public investment or to distribute all the available budget in private shares. For sufficiently high rates of public good survival,  $\delta$ , it is efficient not to invest at all in the public good in period 2, while for low  $\delta$ , the efficient solution has  $I_2^{P*} > 0$ . In any case, our assumptions on  $u$  imply a unique planner solution  $(I_1^{P*}, I_2^{P*})$ .

## 2.2 Bargaining Solution

We model the bargaining process following the classical model of Baron and Ferejohn (1989). In each period, there is a (potentially) infinite number of bargaining stages. At the beginning of each stage, one committee member is chosen at random to make a proposal  $(\{x_{it}\}_{i=1}^{i=n}, g_t)$ , on which all committee members then vote. If a simple majority votes in favor, then the proposal is implemented and the period ends. If it is voted down, then another bargaining stage (within the same period) starts, again with a randomly selected member who submits a proposal, and the process is repeated. There is no discounting between bargaining stages within the same period. A portion of the public investment in period 1 survives until period 2, which creates the link between periods.<sup>5</sup> We focus on symmetric stationary subgame-perfect equilibria with strategies that are anonymous between periods (legislative bargaining equilibrium, hereafter). Given the strict concavity of  $u$ , the legislative bargaining equilibrium is unique in investment levels  $(I_1^{L*}, I_2^{L*})$ .

The equilibrium of a two-period game shares two main features with the one-period game equilibrium: (1) there are no delays on the equilibrium path, as proposals are passed right away; and (2) conditional on public investment being an interior solution, the proposer enjoys a higher private share than any other committee member. As before, we discuss the main forces that govern public investment in each period and refer the reader to Appendix A for the detailed characterization.

Conditional on the public investment in period 1, the maximization problem of a proposer in period 2 involves choosing the cheapest proposal that will pass. There are three alternative routes that the proposer can take. The first route is to invest all available funds in the public good - that is,  $I_2^{L*} = B_2$ . This route is optimal when the stock of the public good that survived from period 1 is sufficiently low, and such proposals pass with a unanimous vote. The second route is to distribute all the available budget in private shares to form a minimum winning coalition. This strategy is optimal when the stock of the public good is sufficiently high. Finally, for intermediate levels of public stock, the proposer's optimal strategy is to invest a portion of the budget in the public good, rewarding  $\frac{n-1}{2}$  randomly chosen members with private shares and appropriating the remaining funds for herself. This interior level of period 2 public investment is characterized by the

<sup>4</sup>Note that if there is no depreciation ( $\delta = 1$ ), then one of the constraints must be binding because the first-order condition for period one reduces to  $u'(I_1^{P*}) = 0$ , which cannot arise.

<sup>5</sup>It is straightforward to see that the analysis presented below generalizes to any quota voting rule, where passage of a proposal requires at least  $q$  supporting votes and  $1 \leq q < n$ .

first-order condition

$$\text{period 2: } \frac{n+1}{2} \cdot u'(\delta I_1^L + I_2^{L*}) = 1. \quad (2a)$$

The comparison between the efficient and bargaining levels of public investment in period 2, when both levels are interior, is instructive (equations (1a) and (2a)). Both the social planner and the member selected to propose an allocation in the bargaining game weigh the marginal benefit of the public investment against its marginal costs. While the marginal cost of public investment is the same in both situations and equal to 1, the marginal benefits are different. The social planner takes into account the fact that a unit of public investment benefits all  $n$  committee members. On the contrary, due to the specifics of the bargaining protocol (majority voting rule), the proposer internalizes the effect on  $\frac{n+1}{2}$  members only - herself and  $\frac{n-1}{2}$  coalition partners. Thus, the bargaining solution underprovides the public good relative to the efficient solution in period 2. This underprovision is purely static, as it is present regardless of the survival rate of the public good  $\delta$  (which is the only dynamic component of our bargaining game). Therefore, we refer to this portion of the underprovision of the public good as the *static inefficiency*.<sup>6</sup>

The proposer selected in period 1 anticipates how her decisions will impact the proposer's choices in period 2 through the accumulation of the public good that is carried over between periods, given the survival rate  $\delta$ . When  $\delta = 0$ , all public investment in period 1 depreciates, and the two-period legislative game becomes simply the one-period legislative game repeated twice. We refer to the game with  $\delta = 0$  as the static game since, in this case, there is no linkage between periods.<sup>7</sup> When  $\delta > 0$ , additional dynamic forces are at play. The first-order condition that characterizes the interior equilibrium investment in period 1,  $I_1^{L*}$ , is:

$$\text{period 1: } \frac{n+1}{2} \cdot \left[ u'(I_1^{L*}) + \frac{dV_2(I_1^L)}{dI_1} \Big|_{I_1^{L*}} \right] = 1, \quad (2b)$$

where  $V_2(I_1^L)$  represents the continuation value of the game at the beginning of period 2 before the proposer has been selected. The left-hand side reflects the distortions from the planner's solution due to both static and dynamic free-rider effects. The first term ( $\frac{n+1}{2} u'(I_1^{L*})$ ) represents the marginal benefit of the public good in period 1 to the proposer's coalition of  $\frac{n+1}{2}$  voters (again, not the social marginal benefit). This is the same static distortion that arises in period 2 and is present regardless of the value of  $\delta$ . The second term captures the *dynamic free-riding* effect because it takes into account how  $I_1^{L*}$  will affect the proposer's (and her coalition partners') continuation value in period 2.

We distinguish between two separate dynamic effects of public investment  $I_1^L$  on the continuation

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<sup>6</sup>From now on, when we use the term inefficiency, we refer to the absolute amount of underinvestment relative to the efficient solution.

<sup>7</sup>Recall that, for our theoretical analysis, we restrict our attention to the stationary subgame-perfect equilibria in which strategies cannot condition on the identities of the committee members in the previous period. When we analyze the data, we will evaluate whether subjects condition their behavior on identities and whether such behavior can be part of an equilibrium.



value  $V_2$ : the direct one, which we refer to as the *durability* effect, and the indirect one, which we call the *crowding-out* effect.

The crowding-out effect arises when period 1 investment is lower than the full budget, and period 2 investment is positive; that is,  $I_1^{L^*} < B_1$  and  $I_2^{L^*}(I_1^{L^*}) > 0$ . In this case, an increase in period 1 investment completely crowds out period 2 investment. The intuition behind this movement is that the period 1 proposer can reduce the side payments to coalition members by increasing  $V_2(I_1^L)$  (by freeing up more period 2 budget for private allocations) and, at the same time, increase her own payoff.

If, in equilibrium, the feasibility constraint in period 2 binds - that is,  $I_2^{L^*}(I_1^{L^*}) = 0$  - then investment in period 1 will not substitute for investment in period 2 at the margin. Hence, in this case, the entire dynamic free-riding effect is due to the direct *durability* effect.

To summarize, the bargaining solution in both periods underprovides the public good relative to the efficient planner's solution. One portion of this underprovision is static since it arises regardless of the ties between periods (captured by parameter  $\delta$  in our game). The other portion, present only in period 1, is dynamic and is increasing with the survival rate  $\delta$ . For low values of  $\delta$ , dynamic inefficiencies are due solely to the crowding-out effect, while for high  $\delta$ , they are due to the durability effect. In the next section, we discuss how one can separate static from dynamic inefficiencies and estimate their magnitudes.

### 2.3 Identification of dynamic inefficiencies

This paper aims to determine whether subjects react to the dynamic aspects of public goods provision. If they do, are their reactions to the static and dynamic free-riding incentives similar to what theory predicts as the parameters of the environment change? Following the theoretical framework presented above, we focus on the distortions in public good provision in period 1 and propose one natural way to disentangle two source of inefficiencies (static versus dynamic), both of which contribute to the low level of public good investment relative to the efficient solution.

Let  $\Delta_1^S$  capture the difference in public good investment in period 1 between the efficient and bargaining solutions when the public good fully depreciates between periods ( $\delta = 0$ ). This is pure static inefficiency since it arises as a result of the proposer taking into account her influence only on the utility of  $\frac{n+1}{2}$  members of the committee in the current period and ignoring the rest of the legislators. Thus,

$$\Delta_1^S = I_1^{P^*}|_{\delta=0} - I_1^{L^*}|_{\delta=0}.$$

When a portion of the public good investment survives between periods - i.e.,  $\delta > 0$  - the difference between the planner's and bargaining investments in period 1 encompasses both static and dynamic inefficiencies. We denote this amount by  $\Delta_1^T$  and refer to it as the total inefficiencies.

$$\Delta_1^T = I_1^{P^*}|_{\delta>0} - I_1^{L^*}|_{\delta>0}.$$

Subtracting the static portion of inefficiencies from the total ones gives us the dynamic ineffi-

ciencies that arise only in the dynamic setup

$$\Delta_1^D = \Delta_1^T - \Delta_1^S.$$

It is straightforward to verify that  $\Delta_1^T(\delta)$  is increasing in  $\delta$ . In other words, the smaller the depreciation of the public good between periods, the larger is the total inefficiency. Since static inefficiencies do not change with  $\delta$ , this means that  $\Delta_1^D(\delta)$  is also increasing in  $\delta$ . Depending on the parameters of the game, this dynamic distortion captures either the crowding-out effect (when  $\delta$  is small) or the durability effect (when  $\delta$  is large).

### 3 Experimental Design

#### 3.1 Parametrization

Our experimental design naturally requires the use of a specific parametric public investment function. In particular, we focus on the power function  $u(g) = 5\sqrt{g}$ .<sup>8</sup> To create the simplest possible environment, which captures all of the forces described in the previous section, we consider committees of three bargainers ( $n = 3$ ) that meet for two consecutive periods. In each period, the committee needs to decide how to allocate a budget of 200 tokens ( $B_1 = B_2 = B = 200$ ) between public good investment and pork to each member of the committee. We conduct all the experiments using the Baron-Ferejohn bargaining protocol described above, and we document participants' behavior in the legislative bargaining game. We then compare this behavior to the theoretical planner's solution to measure inefficiencies that arise from bargaining, using the identification strategy described above.

We conduct three treatments that differ only in the value of  $\delta$ , the survival rate of the public investment. The first treatment has  $\delta = 0$ , and, thus, we refer to it as the static bargaining game and denote it by SB. The other two treatments are dynamic bargaining games, one with a low survival rate,  $\delta = 0.2$  (DB<sup>low</sup>), and one with high survival rate,  $\delta = 0.8$  (DB<sup>high</sup>).<sup>9</sup> These two positive values of  $\delta$  were chosen in a way that allows us to distinguish two types of dynamic inefficiencies: crowding-out and durability effects.

Table 1 displays the predicted values of public investment and private allocations in each period and in each treatment as a percentage of the budget. We also present theoretical values for static, dynamic and total inefficiencies using the planner's solution.<sup>10</sup>

When  $\delta = 0$ , all period 1 inefficiencies are solely static (see the third column under the SB heading). In this case, the planner allocates an extra 15.5% of the budget to investment. When  $\delta$  increases to 0.2 and we move to the DB<sup>low</sup> game, dynamic inefficiencies emerge, and total ineffien-

<sup>8</sup>The choice of this functional form was motivated by the desire to choose a function that is simple and 'familiar' to subjects, as well as easy to describe in the instructions and to present graphically.

<sup>9</sup>Recall that the survival rate of public investment is negatively related to the depreciation rate. That is,  $\delta = 0$  indicates full depreciation;  $\delta = 0.2$  indicates high depreciation; and  $\delta = 0.8$  indicates low depreciation.

<sup>10</sup>The analysis in the theory section shows that dynamic inefficiencies are driven by the period 1 investment decision. For this reason, we measure inefficiencies in Table 1 and derive hypotheses using period 1 investment. Section 4.5 presents the analysis of inefficiencies using aggregate welfare.

cies add up to 27.3% of the budget. In this case, the dynamic inefficiencies are due entirely to the crowding-out effect since  $I_2^{L^*} > 0$ . In terms of magnitude, dynamic inefficiencies account for 11.8% of the budget and 43% of the total inefficiencies. The relative importance of dynamic inefficiencies changes dramatically when we further increase  $\delta$  to 0.8 and move to the  $DB^{\text{high}}$  game. In this game, dynamic inefficiencies are due entirely to the durability effect ( $I_2^{L^*} = 0$ ), and they account for almost 40% of the budget and 72% of total inefficiencies. Although there are differences in magnitude, dynamic inefficiencies represent a substantial and non-negligible amount of total inefficiencies in both  $DB^{\text{low}}$  and  $DB^{\text{high}}$  games.

Table 1: Theoretical Outcomes as % of Budget

	Static Barg SB			Dynamic Barg with $\delta = 0.2$ $DB^{\text{low}}$					Dynamic Barg with $\delta = 0.8$ $DB^{\text{high}}$				
	$P^*$	$L^*$	$\Delta_1^S$	$P^*$	$L^*$	$\Delta_1^T$	$\Delta_1^D$	$\frac{\Delta_1^D}{\Delta_1^T}$	$P^*$	$L^*$	$\Delta_1^T$	$\Delta_1^D$	$\frac{\Delta_1^D}{\Delta_1^T}$
<b>Period 1</b>													
public good $I_1$	28.0	12.5	15.5	44.0	16.7	27.3	11.8	0.43	100	44.9	55.1	39.6	0.72
total private goods $X_1$	72.0	87.5		56.0	83.3				0.0	55.1			
proposer $x_1^{\text{Pr}}$		58.4			55.6					36.8			
coalition member $x_1^{\text{C}}$		29.1			27.7					18.3			
other $x_1^{\text{NonC}}$		0.0			0.0					0.0			
<b>Period 2</b>													
public good $I_2$	28.0	12.5		19.4	9.2				0.0	0.0			
total private goods $X_2$	72.0	87.5		80.6	90.8				100	100			
proposer $x_2^{\text{Pr}}$		58.4			60.5					66.7			
coalition member $x_2^{\text{C}}$		29.1			30.3					33.3			
other $x_2^{\text{NonC}}$		0.0			0.0					0.0			

Notes:  $P^*$  denotes the planner's solution;  $L^*$  denotes Legislative Bargaining equilibrium;  $\Delta^S$  denotes Static Inefficiency;  $\Delta^T$  denotes Total Inefficiencies; and  $\Delta^D$  denotes Dynamic Inefficiency.

We note that the parameters of the game were chosen in a way that gives separation between theoretically predicted investment levels in period 1 in dynamic games (44.9% versus 16.7% of the budget) and, at the same time, results in a similar average expected payment for subjects. The latter property allows us to control for subjects' incentives between treatments, while the former property is important for interpreting the results of the experiments. The consequence of these parameter choices, however, is that total welfare (the sum of period 1's and period 2's welfares) in the legislative bargaining equilibrium is almost identical in two dynamic treatments,  $DB^{\text{low}}$  and  $DB^{\text{high}}$ . Hence, our main focus in this paper will be on period 1 investments.

### 3.2 Experimental Interface and Procedures

We conducted sessions at CASSEL (UCLA) and CESS (NYU) using Multistage software (see Table 2).<sup>11</sup> In each location, we recruited subjects from the general undergraduate pool, and each subject

<sup>11</sup>We find no significant differences in the behavior of subjects at NYU and at UCLA.

participated in, at most, one session. Sessions consisted of 12 or 15 participants.<sup>12</sup> We refer the reader to Appendix B for the instructions that subjects received, screen shots, the detailed script of the practice round, and the quiz that was conducted to make sure that subjects understood the structure of the game and the payoffs.<sup>13</sup>

Table 2: Subjects per Treatment

Treatment	UCLA	NYU
Static Barg (SB)	45 (3 sessions)	
Dynamic Barg with $\delta = 0.2$ (DB <sup>low</sup> )	42 (3 sessions)	
Dynamic Barg with $\delta = 0.8$ (DB <sup>high</sup> )	30 (2 sessions)	24 (2 sessions)

In each session, subjects played ten repetitions of the two-period game; we refer to each repetition as a match. In each match, subjects were randomly assigned to groups of three. We describe here the main features of the interface. To reduce the computational difficulties, subjects saw a graph on the screen depicting how dollars (tokens) invested in the project are converted into payoffs.<sup>14</sup> At the beginning of each period, all subjects were asked to choose how they would distribute the available budget between private allocations and public investment (referred to as the project investment in the instructions). The instructions emphasized how investment in period 1 can generate payoffs in period 2 for dynamic treatments.<sup>15</sup> After all the subjects in a group submitted their proposals, one of the three proposals was selected at random (with equal probability) and presented to all group members for a vote. If the proposal received a majority of votes (at least two out of three), then the period was over. The group then moved on to the second period of the game, in which all subjects were again asked to submit their proposals, and one was chosen at random. If, however, the proposal was rejected in stage 1, then the group remained in the first period, and another bargaining stage started in which all members were asked to submit a new proposal. Throughout the experiment, subjects could follow the full history of the experiment in a box at the bottom of the screen. At the end of the session, one match was selected at random for payment; earnings in that match were divided by ten, and the participants were paid the resulting figure plus the participation fee (\$10) in dollars. Average earnings were approximately \$30, and each session took about two hours.

<sup>12</sup>The two sessions of DB<sup>high</sup> conducted at NYU and one of the DB<sup>low</sup> sessions conducted at UCLA involved 12 subjects. All other sessions involved 15 subjects.

<sup>13</sup>Upon their arrival at the lab, subjects were seated in separate cubicles and handed printed instructions. After all participating subjects entered the lab, the experimenter read the instructions aloud and answered any questions that subjects had. After that, all subjects participated in a practice round, during which the experimenter read the script describing the software interface and showed the slideshow with screenshots. Finally, after the practice round, subjects were asked to answer 16 questions about the rules of the game. Subjects had to answer all of the questions correctly to be able to begin the experiment. This quiz was conducted after the practice round and before the beginning of the paid rounds.

<sup>14</sup>Earlier pilot sessions were conducted with a different (less visual) interface. Those data are available upon request. Although the change to a more visual interface was motivated by our worry that the computational demands were high, there is no clear indication that this affected the results.

<sup>15</sup>The instructions included a table that explains, for investment levels from 0 to 200 (in intervals of 10), how investment in period 1 will translate into period 2 payoffs (see Charness et al. (2004) for an example of the importance of payoff tables). Moreover, subjects were explicitly asked to go over this table when taking the quiz. Subjects in all three treatments received such a table, an example of which is presented in Appendix B.

### 3.3 Experimental Hypotheses

We use three main hypotheses to organize the experimental results. Our first hypothesis highlights the fundamental difference between dynamic and static bargaining games with respect to public investment in periods 1 and 2. While, naturally, public investment is expected to be the same in both periods in the static game, this is not the case in the dynamic game, in which period 1 public investment is predicted to be higher than period 2 public investment. We call this hypothesis the *horizon effect hypothesis* and summarize it as follows.

$$\textit{Horizon effect hypothesis: } I_1^{\text{SB}} = I_2^{\text{SB}} \text{ and } I_1^{\text{DB}^j} > I_2^{\text{DB}^j} \text{ for } j \in \{\text{high}, \text{low}\}.$$

The second hypothesis compares public good investment in period 1 across treatments and asserts that investment in the public good increases with the survival rate of the public good. We refer to this prediction as the *investment hypothesis* and note that it captures another essential feature of the dynamic game - namely, that the benefit of investing in the first period is higher when the depreciation rate is lower.

$$\textit{Investment hypothesis: } I_1^{\text{SB}} < I_1^{\text{DB}^{\text{low}}} < I_1^{\text{DB}^{\text{high}}}.$$

The third hypothesis compares the dynamic portion of inefficiencies that are present in the two dynamic treatments. We refer to this prediction as the *dynamic underprovision hypothesis* and expect the underprovision due to durability effect present in the  $\text{DB}^{\text{high}}$  treatment to be larger than that due to the crowding-out effect present in the  $\text{DB}^{\text{low}}$  treatment.

$$\textit{Dynamic underprovision hypothesis: } 0 < \Delta_1^D |_{\text{DB}^{\text{low}}} < \Delta_1^D |_{\text{DB}^{\text{high}}}.$$

## 4 Aggregate Results

In this section, we present aggregate results. We start by exploring the three hypotheses outlined in Sections 4.2 - 4.4, all of which deal with public investments across periods and across treatments. We continue by exploring the welfare implications of period 1 decisions in Section 4.5, in which we are concerned with the total surplus generated in both periods. In Section 5, we zoom in on the individual behavior of subjects to account for the variation that aggregate data abstract away from. Since the focus of this paper is on public provision in dynamic environments, most of the aggregate and individual results will concern public investments and total welfare. We refer the reader to Appendices C and E, in which we discuss other characteristics of the bargaining process, such as the frequency of delays, the distribution of private allocations between committee members, and the determinants of voting behavior.

## 4.1 Approach to data analysis

In this section, we discuss our approach to data analysis and the statistical tests we use. Unless specified otherwise, we focus on the last five matches in each experimental session in order to reduce noise due to learning behavior. We refer to these matches as *experienced matches*.

When reporting investment levels, we consider two categories of proposals. The first category comprises all proposals submitted by all members of each group in the first stage of period  $t$  for  $t \in \{1, 2\}$ , which we refer to as *all proposals* in period  $t$ . The second category includes proposals that satisfy the minimum winning coalition (MWC) condition, which is defined as proposals in which  $\frac{n-1}{2}$  members of the committee receive a private allocation of no more than 10% of the budget.<sup>16</sup> We refer to this category as *MWC proposals*. The definition of MWC suggests that different members of committees are treated differently with respect to their private allocations: some are included in the coalition, while others are not. Thus, proposals that involve investing the whole budget in the public investment and, therefore, treat all members equally do not satisfy the MWC condition. The subset of proposals that were randomly chosen to be voted on and that received a majority of votes looks very similar to the first category of all proposals and, therefore, is not analyzed separately.<sup>17</sup>

When reporting summary statistics regarding investment levels, we report averages. To compare average investments between different periods, proposal types, and treatments, and to contrast them with theoretical predictions, we use random effects panel regressions with standard errors clustered at the session level. Clustering at the session level accounts for potential interdependencies between observations that come from random re-matching of subjects between matches in a session.<sup>18</sup> Depending on the question under consideration, we use one of the three regression specifications described below to provide statistical statements. In all specifications, the unit of observation is the proposal of a subject in each match.

- Test 1: The first regression specification is used to compare observed public investment with the theoretical predictions described in Section 3.1. We do this by regressing investment levels on a constant term. We report the  $p$ -value of a test where the null hypothesis is that the estimate equals the planner or the legislative bargaining solutions.
- Test 2: The second specification is used to compare investment levels between two groups (whether two treatments or two proposal types). In this case, we regress the quantity of interest, which is investment levels in both groups, on a dummy variable that indicates one of the groups and a constant. We report the  $p$ -value of a test where the null hypothesis is that there is no difference across the two groups.
- Test 3: Finally, to compare average investment levels between two periods of the same treatment, we use a third specification. In this case, for each subject and each match, we construct

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<sup>16</sup>Allowing non-coalition partners to receive small shares is standard in the literature.

<sup>17</sup>Consistent with the previous literature, we find that a majority of proposals are passed without delay in all three treatments and in both periods of the game. These results are presented in Appendix C.

<sup>18</sup>See Fréchet (2012) for a discussion.

a variable that tracks the difference between period 1 and period 2 investment. We regress this difference on a constant and report the  $p$ -value associated with the estimated constant.

We will use the term ‘statistically significant’ when the corresponding null hypothesis can be rejected at the 5% level. Conclusions that we reach based on the analysis of average investment levels remain intact if we use medians instead of averages. These results are largely omitted from the main text of the paper and presented in Appendix D.<sup>19</sup>

## 4.2 Horizon Effect Hypothesis

The two-period bargaining game analyzed in Section 2 is a relatively challenging environment. On the one hand, behavior in the two periods is interdependent, and, on the other hand, behavior is predicted to be different across periods in all but the static treatment. Therefore, we start by assessing whether subjects internalize the fundamental difference between dynamic and static bargaining environments by comparing period 1 and period 2 public investments. According to the horizon-effect hypothesis, period 2 public investment in both dynamic treatments is predicted to be smaller than period 1 public investment. This is true for both the efficient solution and the equilibrium bargaining solution, as depicted in Table 1. The intuition for this result comes from the fact that while the utility of the public good is the same in both periods, the initial stock of the public good in period 2 is at least as high as that in period 1 since  $\delta > 0$ , and public investment in period 1 is non-negative. On the contrary, in the SB treatment, in which  $\delta = 0$ , the public investment in both periods is expected to be the same.

Our test of the horizon-effect hypothesis is based on data presented in Table 3, which depicts average investment levels in each treatment and each period of the game. Consistent with our prediction, public investment is smaller in period 2 than in period 1 in both dynamic treatments. In both cases, the difference is quantitatively large: for instance, including all proposals, period 1 average investment represent 38.7% of the budget compared with 14.4% in period 2 in the DB<sup>low</sup> treatment, while the corresponding fractions are 55.2% in period 1 and 14.1% in period 2 in the DB<sup>high</sup> treatment. Regression analysis (Test 3) confirms that the difference in both treatments is statistically significant ( $p < 0.01$ ).<sup>20</sup> In the static bargaining treatment, the difference is relatively small: 16.7% versus 12.9% for all proposals and 11.1% versus 10.2% for MWC proposals. Regression analysis confirms that period 1 and period 2 investment levels are not statistically significant when we restrict attention to proposals that satisfy MWC in both periods ( $p = 0.30$ ). However, the difference is small but significant when looking at all proposals ( $p = 0.02$ ).

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<sup>19</sup>In Section 4.5, we evaluate the hypotheses using welfare levels instead of investment levels, and we do document some differences between using means and medians. In this case, both measures are reported in the text.

<sup>20</sup>The quantitative differences are also large for MWC proposals. To evaluate whether the difference is statistically significant, we proceed with *Test 3* using two samples. First, we restrict the sample to proposals that satisfy the MWC constraint in both periods. Second, we restrict the sample to proposals that satisfy the MWC constraint in at least one period. In both cases the difference is statistically significant. The corresponding  $p$ -values are  $p = 0.04$  (First sample, DB<sup>low</sup>),  $p < 0.01$  (First sample, DB<sup>high</sup>),  $p < 0.01$  (Second sample, DB<sup>low</sup>), and  $p < 0.01$  (Second sample, DB<sup>high</sup>).

Table 3: Public Investments as % of Budget and Inefficiencies in Experienced Matches

	Static Barg SB				Dynamic Barg with $\delta = 0.2$ DB <sup>low</sup>						Dynamic Barg with $\delta = 0.8$ DB <sup>high</sup>					
	$I_t^{P*}$	$I_t^{L*}$	$I_t$	$\Delta_t^S$	$I_t^{P*}$	$I_t^{L*}$	$I_t$	$\Delta_t^T$	$\Delta_t^D$	$\frac{\Delta_t^D}{\Delta_t^T}$	$I_t^{P*}$	$I_t^{L*}$	$I_t$	$\Delta_t^T$	$\Delta_t^D$	$\frac{\Delta_t^D}{\Delta_t^T}$
<b>Period <math>t = 1</math></b>																
<i>Theory</i>	28.0	12.5		15.5	44.0	16.7		27.3	11.8	<b>0.43</b>	100	44.9		55.1	39.6	<b>0.72</b>
<i>Observed</i>																
<i>all</i>			16.7	11.3			38.7	5.3	-6.0	-			55.2	44.8	33.5	<b>0.75</b>
<i>MWC</i>			11.1	16.9			18.3	25.7	8.8	<b>0.34</b>			29.3	70.7	53.8	<b>0.76</b>
<b>Period <math>t = 2</math></b>																
<i>Theory</i>	28.0	12.5			19.4	9.2					0.0	0.0				
<i>Observed</i>																
<i>all</i>			12.9				14.4						14.1			
<i>MWC</i>			10.2				8.3						6.0			

Notes:  $I_t^{P*}$  is the theoretically predicted efficient level of public investment in period  $t$ .  $I_t^{L*}$  is the theoretically predicted level of public investment in the legislative bargaining solution in period  $t$ .  $I_t$  is the average investment levels as % of Budget for each period in each treatment.  $\Delta^S$  denotes Static Inefficiency.  $\Delta^T$  denotes Total Inefficiencies.  $\Delta^D$  denotes Dynamic Inefficiency. Total, static and dynamic inefficiencies are computed using investment levels reported in this table, as described in Section 2.3. For each period  $t$ , category *all* includes all observed proposals in period  $t$ , while category *MWC* includes period  $t$  proposals that satisfy MWC restriction, as defined in Section 4.1.

**Finding 1:** Aggregate data support the horizon-effect hypothesis and indicate that subjects have a basic understanding of the dynamic tensions in the bargaining environment.

### 4.3 Investment Hypothesis

In this section, we first show that the evidence is in line with the comparative statics of the investment hypothesis and then compare the investment levels with the theoretically predicted ones.

To test the investment hypothesis, we decompose it into three pairwise comparisons and test each separately:  $I_1^{SB} < I_1^{DB^{low}}$ ,  $I_1^{SB} < I_1^{DB^{high}}$  and  $I_1^{DB^{low}} < I_1^{DB^{high}}$ . Furthermore, we test these three inequalities for two categories of proposals, all proposals and MWC proposals, separately, using Test 2 described in Section 4.1. As depicted in Table 3, period 1 investment monotonically increases with the survival rate  $\delta$ , regardless of whether one focuses on all submitted proposals or proposals that satisfy the MWC condition. The regression analysis corroborates this observation. The treatment effect is significant at the 5% level in all pairwise comparisons except for one case, when the significance occurs at the 10% level. Specifically, when considering all submitted proposals, the p-values for the estimated treatment indicator are  $p < 0.01$  for  $I_1^{SB}$  versus  $I_1^{DB^{low}}$ ,  $p < 0.01$  for  $I_1^{SB}$  versus  $I_1^{DB^{high}}$ , and  $p = 0.03$  for  $I_1^{DB^{low}}$  versus  $I_1^{DB^{high}}$ . For MWC proposals, the same p-values are  $p = 0.06$ ,  $p < 0.01$ , and  $p = 0.02$ .

To compare public investments with theoretically predicted ones, we perform a series of tests, the



results of which are summarized in Table 4. A few interesting patterns emerge from these tests. First, public investments are lower than the efficient levels chosen by the benevolent planner in all three treatments, regardless of whether one considers all proposals or only those that involve MWCs - with one exception of all proposals in the first period of DB<sup>low</sup> treatment (first and third rows in Table 4). The comparison between mean investment levels and those predicted by the legislative bargaining solution depends on the proposal category and a treatment, with MWC proposals tracking the legislative bargaining solution more closely in general than all submitted proposals (second and fourth rows in Table 4).

Table 4: Statistical Tests Comparing Investment Levels in Experienced Matches with Planner and Legislative Bargaining Solutions,  $p$ -values

	All proposals					
	Period 1			Period 2		
	SB	DB <sup>low</sup>	DB <sup>high</sup>	SB	DB <sup>low</sup>	DB <sup>high</sup>
$H_0: I_t = I_t^{P^*}$	< 0.01	0.45	< 0.01	< 0.01	0.05	< 0.01
$H_0: I_t = I_t^{L^*}$	0.14	< 0.01	0.03	0.84	0.04	< 0.01
	MWC proposals					
	Period 1			Period 2		
	SB	DB <sup>low</sup>	DB <sup>high</sup>	SB	DB <sup>low</sup>	DB <sup>high</sup>
$H_0: I_t = I_t^{P^*}$	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
$H_0: I_t = I_t^{L^*}$	0.59	0.55	< 0.01	0.03	0.76	< 0.01

**Notes:**  $I_t^{P^*}$  denotes the theoretically predicted efficient level of public investment in period  $t$ , while  $I_t^{L^*}$  denotes the theoretically predicted level of public investment in the legislative bargaining solution in period  $t$ . For each treatment, period and proposal category, we use Test 1 described in Section 4.1 to compare the observed level of public investment with those predicted by theory - namely,  $I_t^{P^*}$  and  $I_t^{L^*}$ .

**Finding 2:** *Investment in the public good increases with the survival rate of the public good, as predicted by the investment hypothesis.*

#### 4.4 Dynamic Underprovision Hypothesis

Table 3 presents the decomposition of period 1 public investment into the static and dynamic components. We use this information to examine the dynamic underprovision hypothesis. There are two parts to this hypothesis. First, dynamic inefficiencies in any dynamic treatment should be positive, which means that total inefficiencies in either dynamic treatment are higher than inefficiencies in the static bargaining treatment. Second, dynamic inefficiencies present in the DB<sup>high</sup> treatment, which are due entirely to the durability effect, are predicted to be higher than those present in the DB<sup>low</sup> treatment, which are due entirely to the crowding-out effect.

Focusing on MWC proposals first, we find evidence in support of all aspects of the hypothesis. First, dynamic inefficiencies in both DB<sup>low</sup> and DB<sup>high</sup> are positive and significantly different than

zero, with corresponding  $p = 0.04$  and  $p < 0.01$ . Second, dynamic inefficiencies in  $DB^{\text{high}}$  are significantly higher than in  $DB^{\text{low}}$  ( $p < 0.01$ ).<sup>21</sup> When we consider all proposals, we still find evidence of higher dynamic inefficiencies in  $DB^{\text{high}}$  than in  $DB^{\text{low}}$  ( $p < 0.01$ ). We also find that dynamic inefficiencies in  $DB^{\text{high}}$  are positive and significantly different than zero ( $p < 0.01$ ). However, there is no evidence of dynamic inefficiencies in  $DB^{\text{low}}$  ( $p = 0.38$ ).

**Finding 3:** *Dynamic inefficiencies due to the durability effect are significant and large in magnitude in the  $DB^{\text{high}}$  treatment, regardless of whether one focuses on all submitted proposals or only on MWC proposals. In the  $DB^{\text{low}}$  treatment, we observe significant dynamic inefficiencies due to the crowding-out effect only for MWC proposals.*

We conclude by noting that the monotonic increase in the dynamic inefficiencies that we establish in this section corresponds to the monotonic increase in the *total* inefficiencies, as well. This is guaranteed by our identification strategy described in Section 2.3 since the total and dynamic inefficiencies differ only by a constant term - the static inefficiencies - which remains the same in all three treatments. Therefore, evidence presented in this section implies that the magnitude of total underprovision across treatments increases with the survival rate  $\delta$ .

## 4.5 Welfare

Up until now, the analysis has focused on period 1 investment decisions, which are a key determinant of the final welfare of a group, as explained in Section 2. Other things equal, a sub-optimal period 1 investment would lead to a welfare loss, as measured by the total surplus generated by a group in both periods. In other words, a dynamic inefficiency in terms of public investment would correspond to a dynamic inefficiency in terms of welfare. However, it is possible that, in period 2, subjects react to sub-optimal period 1 investment in a way that may at least partially offset welfare losses from the period 1 underinvestment. To investigate the possibility that such compensation occurs, we define a welfare measure that incorporates public investments in both period 1 and period 2. The analysis in this section shows that using the welfare measure is qualitatively in line with our previous findings that use period 1 investment.

Define the measure of a group's welfare as the additional surplus generated on top of the minimum possible welfare that a group is guaranteed to achieve, absent any public investments in either period. Specifically, this measure,  $W$ , is defined as:

$$W = \sum_{i=1}^3 x_{i1} + 3u(g_1) + \sum_{i=1}^3 x_{i2} + 3u(g_2) - 2B.$$

In other words,  $W$  adds up payoffs of all three group members in both periods and subtracts  $2B$ , which is the lowest possible total payoff that a group can obtain by distributing all the available budget in private shares and not investing at all in either period.

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<sup>21</sup>To compute these p-values, we use Test 2, in which the dependent variable is the difference between the planner's period 1 investment and actual period 1 investment.

This welfare measure can also be used to break down the total welfare losses into those attributable to static and dynamic sources, in a way that is similar to the earlier analysis of period 1 investment decisions. The results of this alternative approach are presented in the first row of Table 5. First, for each treatment separately, the table presents theoretical values for  $W$  in both the efficient and the legislative bargaining solutions (denoted  $W^{P^*}$  and  $W^{L^*}$ , respectively). Second, the table shows the total welfare loss relative to the planner’s solution ( $\Delta W^T$ ) and decomposes it into static and dynamic components ( $\Delta W^S$  and  $\Delta W^D$ , respectively). The results are very similar to the calculations presented in Table 3, which reports the same analysis, but with respect to period 1 public investments instead of  $W$ . In particular, the underprovision hypothesis can be stated in terms of welfare, instead of period 1 decisions, as  $\Delta W^S < \Delta W^T|_{\text{DB}^{\text{low}}} < \Delta W^T|_{\text{DB}^{\text{high}}}$  or, using the dynamic components, as  $0 < \Delta W^D|_{\text{DB}^{\text{low}}} < \Delta W^D|_{\text{DB}^{\text{high}}}$ .

Before discussing our findings, we should note that a potential drawback from using  $W$  instead of period 1 investment is that it is based on fewer observations, for the following reason. In order to measure  $W$ , one must include period 1 and period 2 proposals, and, hence, only period 1 proposals that were randomly selected (and passed) can be used.<sup>22</sup> For period 1 proposals that were either not selected or failed to pass, one does not observe the period 2 proposals that they would have triggered. Hence, one must drop period 1 proposals that were not selected for a vote or that failed to pass, which represents more than two thirds of the period 1 investment data.<sup>23</sup> One consequence of the lower number of observations is that a few outliers can affect the computation of averages and present a misleading picture. For this reason, our calculations in Table 5 present both means and medians.

Table 5 presents mean and median welfare levels observed in each treatment, as well as their decomposition in terms of static and dynamic sources in the experienced matches. We present data for all proposals separately, as well as only for the proposals that involve minimum winning coalitions in both periods. Consider the underprovision hypothesis, which ranks welfare losses between treatments from the smallest in the SB treatment to the highest in the DB<sup>high</sup> treatment. Using all proposals, the data provide qualitative support of this hypothesis, as the highest average and median levels of welfare losses are observed in the DB<sup>high</sup> treatment, and the lowest average and median levels in the SB treatment.<sup>24</sup> When we restrict our attention to proposals that satisfy the requirements of MWC in both periods, the ordering is preserved for the median and reversed for the mean level of welfare losses in the two dynamic treatments. We note that a few outliers

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<sup>22</sup>If we computed a partial measure using only period 1 proposals, part of the welfare consequences of suboptimal period 1 choices would be missing. More importantly, the missing portion would differ across treatments, as the effect of period 1 choices on period 2 welfare depends on  $\delta$ .

<sup>23</sup>For each period 1 proposal that passed, we can compute  $W$  for all period 2 proposals (those that were selected for a vote and passed and those that were not).

<sup>24</sup>To evaluate whether welfare losses are significantly different across treatments, we use a specification similar to that of Test 2 in Section 4.1, but where the dependent variable is the measure of total welfare loss ( $\Delta W^S$  and  $\Delta W^T$  in static and dynamic treatments, respectively). To evaluate the hypothesis for the median we use a quantile regression. We find that mean total welfare losses in DB<sup>high</sup> are significantly higher than those in the SB treatment using all proposals ( $p = 0.03$ ) or those that satisfy MWCs ( $p < 0.01$ ). The same finding holds for the comparison between the DB<sup>low</sup> and SB treatments for the comparison between means using all proposals ( $p = 0.07$ ) and proposals that satisfy MWCs ( $p < 0.01$ ). For other comparisons, the differences are not statistically significant at standard levels.

Table 5: Welfare Measure: Theory and Observed Outcomes

	Static Barg SB				Dynamic Barg $\delta = 0.2$ DB <sup>low</sup>					
	$W^{P^*}$	$W^{L^*}$	$W$	$\Delta W^S$	$W^{P^*}$	$W^{L^*}$	$W$	$\Delta W^T$	$\Delta W^D$	$\frac{\Delta W^D}{\Delta W^T}$
<i>Theory</i>	112.5	100		12.5	126.6	109.9		16.7	4.2	<b>0.25</b>
<i>Observed</i>										
(a) all										
mean			91.5	21.0			97.6	29.0	8.0	<b>0.28</b>
median			94.2	18.3			107.9	18.7	0.4	<b>0.02</b>
(b) MWC										
mean			92.1	20.4			89.4	37.2	16.8	<b>0.45</b>
median			94.2	18.3			103.5	23.1	4.8	<b>0.21</b>
					Dynamic Barg $\delta = 0.8$ DB <sup>high</sup>					
					$W^{P^*}$	$W^{L^*}$	$W$	$\Delta W^T$	$\Delta W^D$	$\frac{\Delta W^D}{\Delta W^T}$
<i>Theory</i>					201.9	179.4		22.5	9.9	<b>0.44</b>
<i>Observed</i>										
(a) all										
mean							169.4	32.5	11.5	<b>0.35</b>
median							180.1	21.8	3.5	<b>0.16</b>
(b) MWC										
mean							167.0	34.9	14.5	<b>0.42</b>
median							169.6	32.3	14.0	<b>0.43</b>

Notes: In column  $W$ , we report mean and median welfare levels for each treatment.  $W^{P^*}$  and  $W^{L^*}$  depict theoretically predicted welfare levels in the planner and legislative bargaining solutions, respectively. For Theory rows,  $\Delta W^S = W^{P^*}|_{\text{SB}} - W^{L^*}|_{\text{SB}}$  represents predicted static welfare losses; for Data rows,  $\Delta W^S = W^{P^*}|_{\text{SB}} - W|_{\text{SB}}$  represent estimated static welfare losses. For Theory rows,  $\Delta W^T = W^{P^*}|_{\text{DB}} - W^{L^*}|_{\text{DB}}$  represents predicted total welfare losses, while for Data rows,  $\Delta W^T = W^{P^*}|_{\text{DB}} - W|_{\text{DB}}$  represents observed total welfare losses. Finally,  $\Delta W^D = \Delta W^T - \Delta W^S$  represents dynamic welfare losses.

strongly affect the comparison between the means in the two dynamic treatments.<sup>25</sup>

To summarize, findings presented in this section indicate that period 2 public investment decisions do not compensate for the suboptimal public investments in period 1. In other words, measuring the effect of suboptimal period 1 investment using  $W$  leads to qualitative conclusions similar to those we presented when the measure directly uses period 1 public investments.

## 5 Individual Data Analysis

Our previous results considered aggregate-level data. These results suggest a relatively high degree of heterogeneity in public investment decisions across subjects (see, for example, the large difference between average investment levels observed in all versus MWC proposals reported in Table 3). In this section, we look at the individual-level data with the aim of documenting and studying our subjects' main types of strategies in more depth. This section is structured as follows. First, we define three types of strategies and show that these types capture the vast majority of the observed public investments. Then, we document the popularity of each type of strategy and look at their prevalence as the sessions evolve. Second, for each type of strategy, we compare public investments in both periods to the optimal investments conditional on the type of strategy used. Third, we consider payoffs associated with the use of each type of strategy and study whether these payoff differences can account for the differences in the use of the strategies between treatments. Finally, we provide some insights into the rationale for the observed strategies.

### 5.1 Types of strategies and evolution of their use

A strategy in our two-period dynamic game is a proposal in period 1 and a period 2 proposal for each of period 1's possible outcomes. In our data, we partially observe period 2 choices, as we learn each subject's period 2 proposal only for the proposal that passed in period 1. In this section, the unit of observation will be the subject's choice in both periods of the match, and, even though this is an abuse of terminology, we will refer to it as the subject's strategy. There are three types of strategies that account for the vast majority of observed choices:

- *MM strategies*

Involve forming MWCs in both periods (hence the name MM), where MWC proposals are defined as above.

- *EE strategies*

Involve splitting benefits equally among all three members in both periods (hence the name

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<sup>25</sup>An observation involves an MWC if the period 1 and period 2 proposals involve MWCs. This requirement further reduces the dataset in addition to looking only at the period 1 proposals that passed, and, eventually, a few outliers can substantially affect the average. In the  $DB^{\text{low}}$  treatment, there are 67 MWC proposals. Six of these proposals involve no investment in either period, leading to a  $W$  of zero. If these six proposals are excluded, the  $\Delta W^T$  mean moves from 37.2 to 28.4, and the ordering of total welfare losses is again in line with the theoretical predictions. The outliers almost do not affect the median, which moves from 23.1 to 22.5. In terms of statistical tests, the results for the mean depend on the outliers. If the outliers are excluded, there is no significant difference between  $DB^{\text{low}}$  and SB ( $p = 0.34$ ) - including outliers the difference is significant ( $p < 0.01$ ).

EE), where we define as an equal split any proposal in which the difference in private allocations between any two members is not larger than five tokens (2.5% of the budget).<sup>26</sup>

- *EM strategies*

Involve splitting resources equally in period 1 and forming a MWC in period 2 (hence the name EM).

Figure 1 plots the proportion of subjects who submit a proposal of each identified strategy type by match in each treatment. The first observation is that classifying proposals into the three strategy types outlined above accounts for the vast majority of all observed choices: 88% in the SB game and 94% in either of the DB games in the last five matches (see, also, the dotted line that represents the total number of observations that fall into one of the defined types).

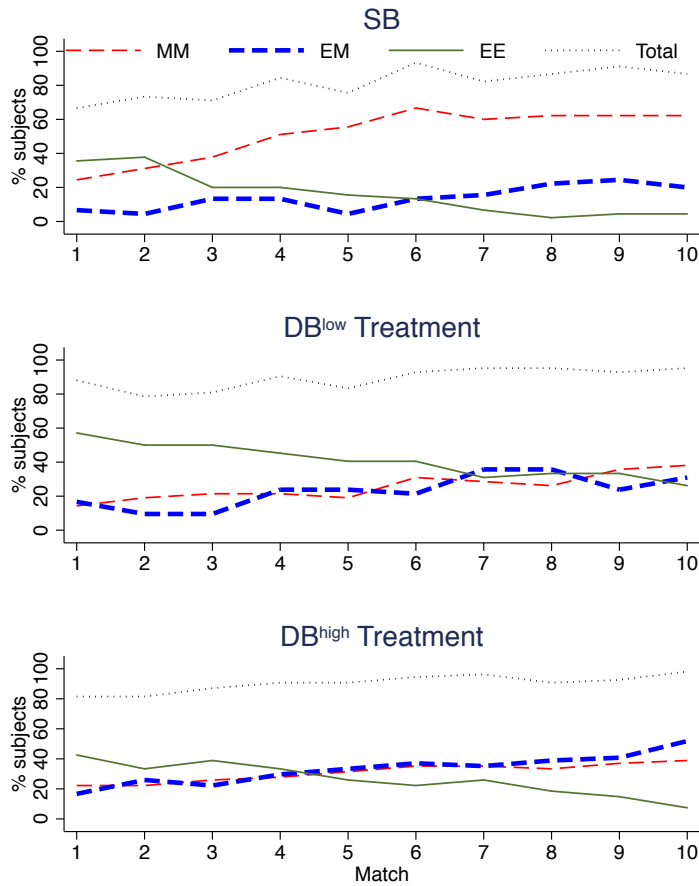


Figure 1: The evolution of strategies used by subjects in each treatment

<sup>26</sup>Formally, let  $x_{it}$  represent the share of the budget corresponding to a private allocation to subject  $i$  at period  $t$ , according to some proposal. We say that the proposal at time  $t$  splits benefits equally (or that all members are included in the proposal) if  $|x_{it} - x_{jt}| \leq 2.5\%$  for all  $i, j$  in the committee. The reason that we allow for small deviations from the exactly equal splits is that the total budget of 200 is not divisible by three.

The relative frequency of each strategy type varies with the treatment and evolves as subjects gain experience with the environment. However, in all three treatments, the frequency of EE strategies decreases as a session evolves, while the frequency of MM and EM strategies increases. Indeed, EE strategies are the most commonly observed type in all treatments at the beginning of the session; however, there is a clear decline in their use by the end of the session, especially in the SB and the DB<sup>high</sup> treatments.<sup>27</sup> The two other types of strategies are observed with increasing frequency over time. In the SB game, by the end of the session, two thirds of all strategies are of type MM, while in the DB treatments, about one third of all proposals are of type MM. Type EM strategies are relatively more common in the DB treatments, accounting for 30%-40% of all proposals in the last five matches, which we refer to as experienced matches.

An analysis of the transitions between strategies reveals that subjects adopting MM strategies in the static treatment and MM or EM strategies in the DB<sup>high</sup> treatment are very likely to stick with this type of strategy. Conditional on changing the type of strategy used, the most frequent transitions are from EM to MM and from EE to EM. A detailed analysis of the transitions between strategies is discussed later in this section.

## 5.2 Behavior within each strategy type

Each of the three types of strategies identified above entails very different public investment and allows, in principle, for heterogeneous behavior within the category. For example, for the MM strategy, infinitely many proposals satisfy the MWC condition in both periods; yet, for all such proposals, the bargaining equilibrium identifies one as optimal behavior given the imposed restrictions of symmetry, stationarity and anonymity between periods. Similarly, conditional on using the EE strategy, optimal behavior involves choosing the efficient level of public investment and distributing the remaining funds equally.<sup>28</sup> Finally, the EM strategy combines elements of the bargaining equilibrium and the efficient planner's solution. Thus, conditional on using the EM strategy, optimal behavior involves choosing public provision at the efficient level in period 1 and following the prescription of the bargaining equilibrium for public provision in period 2.

Given the prevalence of all three types of strategies in our data, the first natural question is whether, conditional on using a particular type of strategy, subjects choose investment levels that are close to the theoretical ones described above. To address this, we calculate the theoretically optimal proposals within each category and compare them to the observed proposals, conditional on the type of strategy used. The results are presented in Table 6.<sup>29</sup>

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<sup>27</sup>The decline in frequency of the EE strategy over time is reminiscent of: 1) In Fréchette et al. (2003), the authors observe that distributions offering an equal division of payoffs decrease in popularity with experience. 2) In VCM experiments, contributions decline with experience - i.e., outcomes become less efficient over time.

<sup>28</sup>Notice that the EE strategy cannot be supported as the sub-game perfect equilibrium in a finite period dynamic bargaining game, as treating committee members equally in the last period is not optimal.

<sup>29</sup>To compare observed investment levels with those predicted by theory, we use regression analysis similar to *Test 1* described in Section 4.1. We use random effects panel regressions to compare average investment levels with those predicted by theory. Table 13 in Appendix E reports the corresponding median investment levels.

Table 6: Investment as % of Budget in Experienced Matches

Treatment	Type MM			Type EM			Type EE		
	theory	mean	$p$ -value	theory	mean	$p$ -value	theory	mean	$p$ -value
<b>Period 1</b>									
SB	12.5	11.4	0.86	28.0	28.5	0.49	28.0	31.1	< 0.01
DB <sup>low</sup>	16.7	18.6	0.54	44.0	50.7	0.51	44.0	49.5	0.67
DB <sup>high</sup>	44.9	30.1	< 0.01	100	71.0	0.03	100	74.2	< 0.01
<b>Period 2</b>									
SB	12.5	10.3	0.01	12.5	9.6	0.13	28.0	31.1	< 0.01
DB <sup>low</sup>	9.2	9.2	0.72	9.2	8.1	0.97	19.3	24.9	0.09
DB <sup>high</sup>	0.0	6.6	< 0.01	0.0	5.4	0.02	0.0	40.3	< 0.01

Notes: We run a regression in which we regress investment levels on a constant. We use random effects panel regressions clustering standard errors by session. For each type of proposal and each treatment, we report  $p$ -value from a test where the null hypothesis is that the estimate equals the theoretically predicted one.

Table 6 reveals several interesting patterns. First, investment levels are very different between proposals of different types. In particular, subjects using EM or EE strategies invest a substantially higher share of resources in the public good in period 1 than those that use MM strategies. Moreover, within each category, public investment in period 1 increases monotonically with the survival rate  $\delta$ .<sup>30</sup> Finally, in SB and DB<sup>low</sup> treatments, for each of the three types of strategies, public investments track theoretically optimal levels closely in both periods.<sup>31</sup> On the contrary, in the DB<sup>high</sup> treatment, we observe underprovision of the public good in period 1 and overprovision of the public good in period 2 relative to the conditional optimal levels, irrespective of the type of strategy used by the subjects. Notice that in the DB<sup>high</sup> treatment, in all but one case, the conditionally optimal public investment is a corner solution (the exception is optimal period 1 public provision for MM strategy). Therefore, any deviations due to mistakes and learning will necessarily be in the direction of underinvesting in period 1 and overinvesting in period 2, which is precisely what our data suggest.

### 5.3 Payoffs for each type of strategy and transitions between strategy types

Table 7 displays information on payoffs by treatment, period and strategy type. For each proposal, we compute the payoffs for the proposer and for the two non-proposers, who we refer to as non-proposer  $i$  and non-proposer  $j$ . Whenever a proposal involves MWC, we assign the label non-proposer  $i$  to a member who is included in the coalition and the label  $j$  to the remaining non-proposer. The table reports the theoretical prediction, the mean (using data from the experienced matches) and a  $p$ -value of a test in which the null hypothesis is that the prediction is equal to the mean. Further, Table 8 shows how our subjects transition between strategy types after gaining experience with the game. Specifically, for each subject, we fix the strategy type that they select in

<sup>30</sup>Within each category, the difference in average investment is significant at least at the 5% level in all but two cases. For proposals of type MM, when comparing the SB and the DB<sup>low</sup> treatments, and for proposals of type EE, when comparing the DB<sup>low</sup> and the DB<sup>high</sup> treatments, the difference is significant at the 10% level. For these statements, we use the specification in Test 2 (see section 4.1).

<sup>31</sup>The differences are relatively small, but in some cases statistically significant (e.g., period 1 investment for type EE proposals in SB.)



Table 7: Payoffs in Tokens in All Submitted Proposals in the Experienced Matches

	Type MM			Type EM			Type EE		
	theory	mean	<i>p</i> -value	theory	mean	<i>p</i> -value	theory	mean	<i>p</i> -value
<b>Period 1</b>									
<i>SB</i>									
Proposer	141.8	113.6	<0.01	85.4	84.9	0.15	85.4	84.7	<0.01
Non-proposer <i>i</i>	83.2	108.2	<0.01	85.4	84.9	0.11	85.4	84.6	<0.01
Non-proposer <i>j</i>	25.0	23.9	0.73						
<i>DB<sup>low</sup></i>									
Proposer	140.1	106.1	<0.01	84.3	80.1	0.05	84.3	80.3	0.05
Non-proposer <i>i</i>	84.5	104.9	<0.01	84.3	79.9	0.06	84.3	80.1	0.06
Non-proposer <i>j</i>	25.0	24.7	0.64						
<i>DB<sup>high</sup></i>									
Proposer	121.0	106.2	<0.01	70.7	77.3	<0.01	70.7	76.6	<0.01
Non-proposer <i>i</i>	84.2	104.6	<0.01	70.7	77.3	<0.01	70.7	76.5	<0.01
Non-proposer <i>j</i>	47.4	36.4	<0.01						
<b>Period 2</b>									
<i>SB</i>									
Proposer	141.8	114.1	<0.01	141.8	109.4	<0.01	85.4	84.5	<0.01
Non-proposer <i>i</i>	83.2	107.6	<0.01	83.2	108.1	<0.01	85.4	84.5	<0.01
Non-proposer <i>j</i>	25.0	22.8	0.18	25.0	19.4	0.04			
<i>DB<sup>low</sup></i>									
Proposer	145.2	114.0	<0.01	150.2	116.1	<0.01	91.3	88.8	<0.01
Non-proposer <i>i</i>	85.6	112.7	<0.01	90.6	115.4	<0.01	91.3	88.6	<0.01
Non-proposer <i>j</i>	25.0	22.7	0.06	30.0	25.1	0.06			
<i>DB<sup>high</sup></i>									
Proposer	175.8	140.0	<0.01	196.4	148.0	<0.01	129.8	106.6	<0.01
Non-proposer <i>i</i>	109.0	138.1	<0.01	129.8	147.1	<0.01	129.8	106.2	<0.01
Non-proposer <i>j</i>	42.4	45.6	0.09	63.2	53.5	<0.01			

Notes: Exchange rate: 10 tokens = \$1. Whenever the proposal involves MWC, then non-proposer *i* is a member who is included in the coalition, while non-proposer *j* is the other non-proposer, who is excluded from the coalition. We run a regression in which we regress payoffs on a constant, while clustering standard errors by session. We use random effects panel regressions, and for each type of proposal and each treatment, we report the *p*-value from a test in which the null hypothesis is that the estimate equals the theoretical prediction.

match 6 and compute the likelihood that they select each possible type in the last four matches of the session.

Figure 1 and Table 8 show that the popularity of type EE strategies decreases as the session evolves. The information on payoffs suggests why this is the case. Indeed, period 2 payoffs for proposers are lowest for those submitting type EE strategies, and experiencing low period 2 payoffs can disincentivize the subject from using this strategy type.

On the contrary, both type MM and type EM strategies feature high persistence, as can be seen from Table 8. The payoffs presented in Table 7 provide a rationale for this pattern. We do observe some differences in payoffs across these two strategy types: Payoffs for type MM strategies are higher for those in the coalition, while type EM strategies allow for higher period 2 payoffs given the relatively higher investment in period 1 (most notably in the  $DB^{\text{high}}$  treatment). However, differences between strategies are small if we compute total payoffs, adding period 1 and period 2. Consider the  $DB^{\text{high}}$  treatment. The payoff of a type MM strategy is 190 tokens (before the identity

of the proposer is revealed), which is only slightly below the 193 tokens corresponding to a type EM. Thus, for the two strategy types that correspond to most of our data in later matches, the difference in payoffs is relatively small.<sup>32</sup> This suggests that the choice between type MM and type EM strategies is not based on a difference in payoffs. In the next section, we explore a rationale for selecting type EM strategies.

Table 8: Transitions from Strategy Types used in Match 6 to Latter Matches

Treatment	Type in Match 6	Prob. selects each strategy type in matches 7-10 (in %)			
		Type MM	Type EM	Type EE	Other
SB	Type MM	82.5	13.3	0.0	4.2
	Type EM	20.8	42.5	8.3	12.5
	Type EE	8.3	29.2	25.0	37.5
	Other	41.7	0.0	0.0	58.3
DB <sup>low</sup>	Type MM	71.2	9.6	9.6	9.6
	Type EM	36.1	61.1	0.0	2.8
	Type EE	2.9	27.9	64.7	4.4
	Other	16.7	58.3	25.0	0.0
DB <sup>high</sup>	Type MM	77.6	10.5	6.6	5.3
	Type EM	13.8	78.8	7.5	0.0
	Type EE	4.2	37.5	52.1	6.3
	Other	50.0	8.3	0.0	41.7

#### 5.4 A rationale for type EM strategies

To illustrate the rationale, consider an EM strategy that involves public investment at the planner's level in period 1 and an equal division of the remainder. In period 2, the proposer behaves just as in the bargaining equilibrium (forms an MWC and invests in the public good optimally given period 1 investment), except for the choice of the coalition partners. If period 1's proposer provides an amount of public good at the efficient level, then the proposer in period 2 invites period 1's proposer into the coalition in period 2. Otherwise, the period 2 proposer punishes period 1's proposer by excluding her from the coalition in period 2. It is feasible to implement such strategies in our experiment since the ID numbers of the committee members remain the same within a match. Theoretically, however, such a strategy cannot be supported as a sub-game perfect equilibrium since the rewards and punishments are not credible. In the period 2 subgame, after a period 1 proposer deviates from *cooperation* (efficient investment in period 1), any other committee member should exclude her from the coalition. Because she is not included in coalitions proposed by others, the punished agent's continuation value is the lowest among all agents. Thus, any proposer is tempted to deviate, pay her less than the alternative, and include her in the coalition.

To see whether the described punishments/rewards mechanism is consistent with the behavior of subjects that opted to use the EM strategy, we focus on period 2 proposals from subjects who were *not* proposers in period 1. Let  $x_2^{P1}$  be the private allocation in period 2 to a subject who was the

<sup>32</sup>In Table 16 of Appendix E, we explore this comparison further.

Table 9: Punishments: Private allocation to period 1 proposer ( $x_2^{P1}$ )

Strategy type	Period 1 Proposer	SB	DB <sup>low</sup>	DB <sup>high</sup>
$EM_{\text{strategy}} = 1$	$A1 = 1$	21.66	27.78	29.05
	$A1 = 0$	14.14	10.45	19.55
$EM_{\text{strategy}} = 0$	$A1 = 1$	23.10	21.31	19.70
	$A1 = 0$	20.50	19.52	22.29

proposer in period 1 (P1). The dummy variable  $A1$  takes value 1 if the period 1 proposer proposed an allocation that equally benefits all three members and 0 otherwise. A period 2 proposer that uses the EM strategy punishes the period 1 proposer by allocating her  $x_2^{P1} = 0$  whenever  $A1 = 0$  and rewards her by  $x_2^{P1} > 0$  whenever  $A1 = 1$ . Therefore, we would expect  $E(x_2^{P1}|A1 = 0) < E(x_2^{P1}|A1 = 1)$ . In contrast, we would expect to observe no such difference for other types. We will test for this hypothesis by estimating for each treatment:

$$x_2^{P1} = \alpha_0 + \alpha_1 \cdot A1 + \alpha_2 \cdot EM_{\text{strategy}} + \alpha_3 \cdot (A1 \times EM_{\text{strategy}}) + \epsilon,$$

where  $\epsilon \sim N(0, \sigma)$  and  $EM_{\text{strategy}}$  is a dummy variable that takes value 1 if the proposal involving  $x_2^{P1}$  is of Type EM (as defined in Section 5.1). We estimate a random effects regression for each treatment separately and report in Table 9 the average private allocations to period 1 proposers, depending on their period 1 behavior ( $A1$ ) and the type of proposal in period 2.<sup>33</sup> Notice that for proposals that are not of the EM type ( $EM_{\text{strategy}} = 0$ ), there is no quantitative difference between proposers who cooperated ( $A1=1$ ) and those who did not ( $A1 = 0$ ). The difference is dictated by the estimates of  $\alpha_2$ , which are not significant for DB treatments and, although significant, are relatively small in the SB case. This is no longer the case for Type EM proposals. Differences are significant in the DB treatments, but of similar magnitude in all cases, approximately between 10% and 15% of the budget.

The previous evidence shows that subjects using EM strategies offer lower payoffs to period 1 proposers who deviate from cooperation, but they still offer a positive amount, on average, while the theory predicts a payoff of zero. To inspect this further, Figure 2 presents the distribution of  $x_2^{P1}$  for DB treatments. Consider the top left graphs summarizing the information for EM strategies in the DB<sup>low</sup> treatment. When the proposer does not cooperate in period 1 ( $A1 = 0$ ), the mode involves zero private allocations. In contrast, when the proposer cooperates in period 1 ( $A1 = 1$ ), there is a large mass with positive private allocations. The same qualitative finding holds for EM proposals in the DB<sup>high</sup> treatment. This pattern is no longer observed if we focus on MM proposals in both dynamic treatments (second row). The mass of zero offers does not show significant differences, depending on the behavior of period 1's proposer ( $A1 = 0$  versus  $A1 = 1$ ).

<sup>33</sup>The estimated coefficients from the equation above are reported in Table 17 of Appendix E.

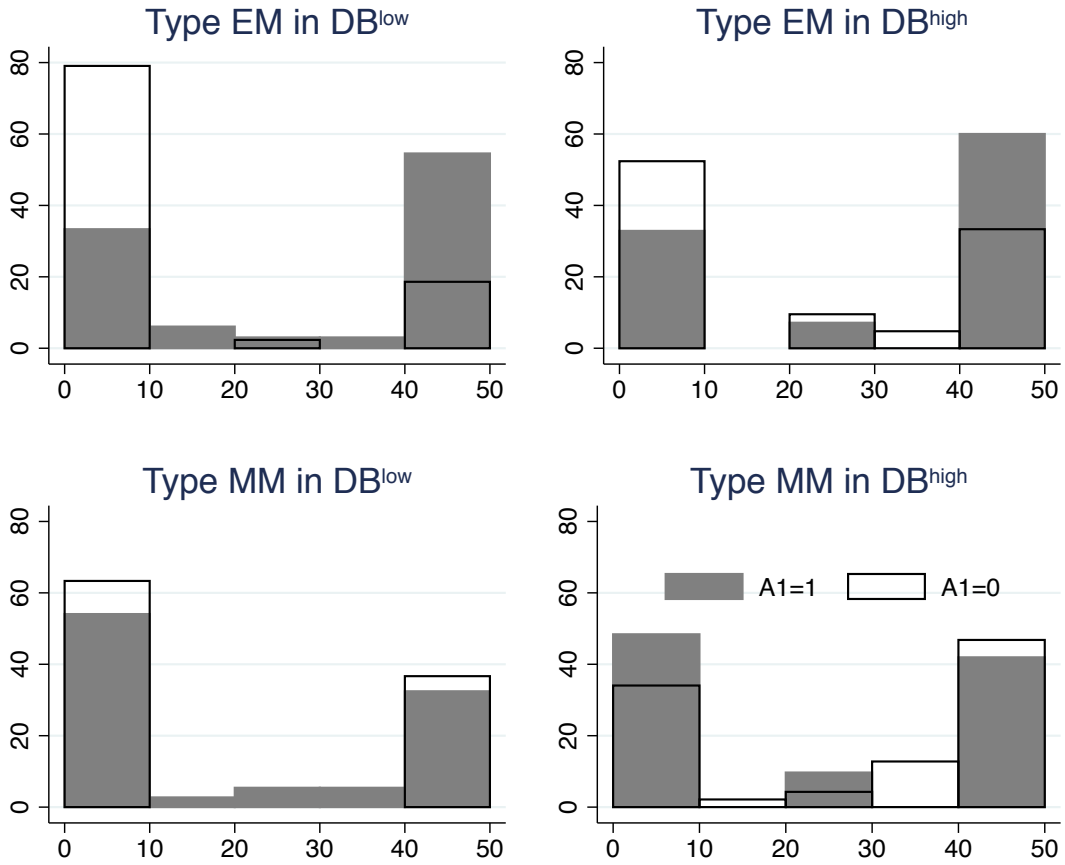


Figure 2: Distribution of Private Period 2 Allocations to Period 1 Proposers as % of Budget

Notes: This figure includes period 2 proposals from subjects who were not proposers in period 1.

## 6 Conclusion

When there are dynamic linkages in the inter-temporal provision of a durable public good, the usual static inefficiencies are present, but new, dynamic inefficiencies arise. Since introducing a dynamic link in a model is typically more theoretically demanding, a prominent question is whether such inefficiencies are empirically meaningful. In this paper, we design and report the findings of an experiment that can isolate static and dynamic inefficiencies in a two-period laboratory environment.

When the theory predicts dynamic inefficiencies to be large, the data are in line with the prediction. Our data indicate that subjects respond to the incentives in the environment similarly to the subjects in earlier experiments on legislative bargaining. With experience, bargaining delays become infrequent, and minimum winning coalition proposals become more prevalent. The main focus of our analysis is on period 1 investment behavior because that is key to identifying dynamic inefficiencies and measuring the extent to which they affect outcomes. On average, investment in the first period is highest when the proportion of period 1 investment that survives in period 2 is high - i.e., the depreciation rate of the durable public good is low. Moreover, period 1 investment is monotonically lower as the depreciation rate increases. Accordingly, when dynamic linkages are relatively low, dynamic inefficiencies become less important, as the theory predicts. Furthermore, we identify two sources of dynamic inefficiencies - the durability effect and the crowding-out effect - and estimate their magnitudes. Our data indicate that, consistent with the theory's predictions, underprovision due to the durability effect is larger than that due to the crowding-out effect; however, the magnitudes of both effects are lower than theory predicts. Overall, our data indicate that dynamic inefficiencies can be empirically quite large, especially when depreciation rates are low.

We also document heterogeneity in individual behavior. Some subjects propose investment levels that are close to the planner's in both periods, especially in early matches. While those subjects may be driven by altruism, the prevalence of this behavior is substantially reduced as the session evolves. In later matches, we are able to identify two canonical types of behavior that capture most of the data. First, a large proportion of subjects use strategies that involve minimum winning coalitions and display investment levels that are, on average, close to the theoretical predictions. The second prevalent behavior involves strategic cooperation, which is seen among subjects whose proposals in the first period are significantly higher than the theoretical level, and who then use a minimum winning coalition in period 2 to reward/punish period 1 choices. According to such behavior, if the investment level proposed in period 1 was efficient (or nearly efficient), then the committee member who made that proposal would be invited into the coalition in period 2 and excluded otherwise. While this behavior is not sub-game perfect, average payoffs are quite close to those for subjects who always proposed a minimum winning coalition. This behavior suggests that a non-negligible fraction of subjects condition period 2 behavior on period 1 outcomes.

We close with two comments about the two-period approach that we use to study dynamic free-riding. First, while much of the theory about the dynamic provision of durable public goods has been studied using infinite horizon models, the basic phenomenon of dynamic free-riding appears even in simple finite horizon models with two periods of public good accumulation. This allows for much

simpler experimental designs, compared with experiments that are designed to mimic an infinite horizon using random termination rules. Methodologically, it allows for a more straightforward analysis of the data, which always appear in blocks of two periods, as opposed to the data from experiments with random termination rules, where different observations have different numbers of periods. On the other hand, the two-period models lack the elegance of the infinite horizon models and cannot address deeper theoretical issues about convergence to stationary states (public good levels) and Markov equilibrium. The philosophy behind our design is that this tradeoff probably favors the two-period model if the goal of the experiment is to compare treatments aimed at sorting out and identifying static and dynamic free-riding effects. In other contexts, where the goal is to evaluate outcomes relative to the predicted long-run steady states or to test for Markov equilibrium, the tradeoff tilts the other way, and there are several examples of this. See, for example, Battaglini and Palfrey (2012), Battaglini et al. (2012), Vespa (2015), and Battaglini et al. (2013).

Second, other dynamic environments have been modeled theoretically using infinite horizon stochastic games, which may also merit study in the laboratory using a two-period approach. An example of this is Battaglini et al. (2014)'s experimental study of a simple two-period version of the Battaglini and Coate (2008) infinite horizon model of the political economy of debt and public good provision.

The study of dynamic games in the lab is still relatively new. Although questions having to do with public goods or legislative bargaining have a long history in experimental economics, how this knowledge can be translated to dynamic environments is not self-evident. We think that a richer and more nuanced understanding of the impact of dynamic linkages on these environments can be obtained by studying both infinitely repeated games and simpler games that allow the researcher to focus better on certain features of the dynamic environment. In the context of legislative bargaining with durable public goods, we show that, indeed, subjects react to the tensions identified by the model - in particular, the force of both static and dynamic free-riding. However, we also find that many subjects deviate from the equilibrium strategy in favor of an efficient strategy with an easy-to-implement punishment.

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## A Proofs

### A.1 The Efficient Solution

The maximization problem specified in Section 2.1 can be re-written as

$$\max_{I_1^P, I_2^P} [B_1 - I_1^P + n \cdot u(I_1^P) + B_2 - I_2^P + n \cdot u(\delta I_1^P + I_2^P)] \text{ s.t. } 0 \leq I_1^P \leq B_1 \text{ and } 0 \leq I_2^P \leq B_2.$$

There are several cases to deal with, depending on which, if any, constraints are binding. If no constraints are binding, then there is an interior solution,  $(I_1^{P*}, I_2^{P*})$  characterized by two first-order conditions:

$$\begin{aligned} u'(\delta I_1^{P*} + I_2^{P*}) &= \frac{1}{n} \\ u'(I_1^{P*}) + \frac{\delta}{n} &= \frac{1}{n} \end{aligned} \tag{1}$$

If the solution is not interior, the constraints can be binding in several ways. One possibility is that  $I_1^P \leq B_1$  is binding. A second possibility is that  $0 \leq I_2^P$  is binding. A third possibility is that  $I_2^P \leq B_2$  binds, but this is an uninteresting case and, in the rest of the paper, we assume that it never binds. Notice that the constraint  $0 \leq I_1^P$  is never binding because of the Inada condition on  $u$ . The constraint  $0 \leq I_2^P$  is binding when the value of  $I_1^{P*}$  that solves (1) is such that  $u'(\delta I_1^{P*}) < \frac{1}{n}$ , which happens if  $\delta$  is sufficiently large. In this case, as long as  $I_1^P \leq B_1$  is not also binding, the solution is given by:

$$u'(I_1^{P*}) + \delta u'(\delta I_1^{P*}) = \frac{1}{n}, \tag{2}$$

the second equation of (1), with  $I_2^{P*} = 0$ . If both  $I_1^P \leq B_1$  and  $0 \leq I_2^P$  bind, then the solution is  $I_1^{P*} = B_1$ ,  $I_2^{P*} = 0$ . If only  $I_1^P \leq B_1$  is binding, then the solution is given by the first equation of (1), with  $I_1^{P*} = B_1$ . Finally, observe that our assumptions on  $u$ , imply a unique planner solution  $(I_1^{P*}, I_2^{P*})$ .

In our experiments, the utility of the public good is given by  $u(g) = Ag^\alpha$ . The interior efficient solution depends on the value of  $\delta$ :

$$\text{if } \delta^{1-\alpha} < 1 - \delta \text{ then } \begin{cases} I_1^{P*} = \left[ \frac{1 - \delta}{nA\alpha} \right]^{\frac{1}{\alpha-1}} \\ I_2^{P*} = \left[ \frac{1}{nA\alpha} \right]^{\frac{1}{\alpha-1}} - \delta \left[ \frac{1 - \delta}{nA\alpha} \right]^{\frac{1}{\alpha-1}} \end{cases}$$



$$\text{if } \delta^{1-\alpha} \geq 1 - \delta \text{ then } \begin{cases} I_1^{P*} = \left[ \frac{1}{nA\alpha(1 + \delta^\alpha)} \right]^{\frac{1}{\alpha-1}} \\ I_2^{P*} = 0 \end{cases}$$

## A.2 The Bargaining Equilibrium

### A.2.1 Period 2

The randomly chosen proposer needs only to gain the support of  $\frac{n-1}{2}$  other members of the committee. Denote by  $x_2^{\text{Pr}}$  the private allocation that the proposer will keep for herself and by  $x_2^{\text{C}}$  the amount she will give to  $\frac{n-1}{2}$  non-proposer committee members ‘in her coalition.’ Then, her maximization problem is given by:

$$\begin{aligned} & \max_{(x_2^{\text{Pr}}, x_2^{\text{C}}, I_2^L)} [x_2^{\text{Pr}} + u(g_2)] \\ \text{s.t. } & \begin{cases} x_2^{\text{Pr}} + \frac{n-1}{2} \cdot x_2^{\text{C}} + I_2^L \leq B_2 \\ x_2^{\text{C}} + u(g_2) \geq V_2(I_1) \\ 0 \leq I_2^L, 0 \leq x_2^{\text{Pr}}, 0 \leq x_2^{\text{C}} \\ g_2 = \delta I_1^L + I_2^L, \end{cases} \end{aligned}$$

where  $I_1^L$  is the level of public good implemented in period 1 of the legislative bargaining game, and  $V_2(I_1^L)$  is the value of the game in the second period, as a function of  $I_1^L$ , before a proposer has been selected. The first constraint involves re-writing the budget constraint using the symmetry assumption. In other words, the private allocation for the proposer, plus an equal amount assigned to each other member of a minimum winning coalition (MWC), cannot be higher than the available funds after investment ( $B_2 - I_2^L$ ), where subscript  $L$  stands for the legislature. The second constraint guarantees the participation of other coalition members. A non-proposer who is included in the coalition will vote in favor of the proposal if the utility he gets from it (LHS) is at least as high as the equilibrium expected value of rejecting it ( $V_2$ ). The remaining constraints are feasibility constraints. Since it is a strictly concave problem, there will be a unique solution for the equilibrium period 2 investment level,  $I_2^L$ . Assuming an interior solution,  $0 \leq I_2^L \leq B_2$ , it is characterized by:

$$u'(\delta I_1^L + I_2^L) = \frac{2}{n+1}. \quad (3)$$

In period 2, the proposer weighs the marginal benefit to the public good *to the coalition of  $\frac{n+1}{2}$  voters* against the marginal cost in units of private good of investing an extra unit in the public good.

FOC (3) captures the optimal period 2 investment in the bargaining game as a function of the investment in the first period,  $I_1$ . As in the analysis of the planner’s solution, it is possible that  $u'(\delta I_1^L) < \frac{2}{n+1}$ , in which case (3) violates the constraint  $0 \leq I_2^L$ . Thus, the full characterization of

how  $I_2^L$  varies as a function of  $I_1^L$  is the following:

$$I_2^L(I_1^L) = \begin{cases} u'^{-1} \left[ \frac{2}{n+1} \right] - \delta I_1^L & \text{if } u'(\delta I_1^L) \geq \frac{2}{n+1} \\ 0 & \text{otherwise} \end{cases}.$$

The funds remaining once the unique optimal investment level has been determined are simply  $B_2 - I_2^L(I_1^L)$ . These remaining funds will be allocated among committee members just as in a Baron-Ferejohn multilateral bargaining game with no public good, giving:

$$\begin{aligned} x_2^C &= \frac{1}{n} (B_2 - I_2^L(I_1)) \\ x_2^{\text{Pr}} &= \frac{n+1}{2n} (B_2 - I_2^L(I_1)). \end{aligned}$$

Finally, we can also use the equilibrium levels of these allocations to compute the equilibrium continuation value in period 2,  $V_2(I_1)$ :

$$V_2(I_1^L) = \frac{1}{n} (B_2 - I_2^L(I_1^L)) + u(\delta I_1^L + I_2^L(I_1)). \quad (4)$$

### A.2.2 Period 1

The selected period 1 proposer anticipates how her decisions will impact choices in period 2. The maximization problem of the proposer in period 1 can be written as:

$$\begin{aligned} & \max_{(x_1^{\text{Pr}}, x_1^C, I_1^L)} [x_1^{\text{Pr}} + u(I_1^L) + V_2(I_1^L)] \\ & \text{s.t.} \quad \begin{cases} x_1^{\text{Pr}} + \frac{n-1}{2} x_1^C + I_1^L \leq B_1 \\ x_1^C + u(I_1^L) + V_2(I_1^L) \geq V_1 \\ 0 \leq I_1^L \leq B_1, 0 \leq x_1^{\text{Pr}}, 0 \leq x_1^C \end{cases}, \end{aligned}$$

where  $V_1$  is the expected value of the game to each player before a proposer has been selected. The function to maximize includes the proposer's period 1 utility and the equilibrium expected value of the game for period 2, which depends on  $I_1^L$ . Maximization is constrained by the budget and by the fact that any coalition member expects the proposal to provide at least as much as he would receive by rejecting it ( $V_1$ ). The first constraint obviously holds with equality. If the second constraint is binding and the feasibility constraints are not binding (i.e., the solution is interior), then the maximization problem for the proposer in period 1 can be rewritten as:

$$\max_{I_1^L} \left[ B_1 - I_1^L - \frac{n-1}{2} V_1 + \frac{n+1}{2} [u(I_1^L) + V_2(I_1^L)] \right]$$

The first-order condition that characterizes the equilibrium investment in period 1,  $I_1^L$  is:

$$u'(I_1^L) + \frac{dV_2}{dI_1} \Big|_{I_1^L} = \frac{2}{n+1}. \quad (5)$$

The left-hand side again reflects the distortions from the planner's solution due to a combination of free-rider effects (both dynamic and static) and the bargaining advantage of the period 1 proposer. There are two separate dynamic effects because  $I_1^L$  affects  $V_2$  in two ways: first, there is a direct effect on the level of public good in period 2, which we refer to as the *durability effect*; second, there is an *indirect* effect on the equilibrium private allocations in period 2, which we call the *crowding-out* effect.

$$\frac{dV_2}{dI_1} \Big|_{I_1^L} = -\frac{1}{n} \frac{dI_2^L}{dI_1} \Big|_{I_1^L} + \left[ \delta + \frac{dI_2^L}{dI_1} \Big|_{I_1^L} \right] u' [\delta I_1^L + I_2^L(I_1^L)] \quad (6)$$

**Case 1: Interior solution.** At an interior solution (i.e.,  $I_1^L < B_1$  and  $I_2^L(I_1^L) > 0$ ),  $\frac{dI_2}{dI_1} = -\delta$ , so the first term reduces to  $\frac{\delta}{n}$ , and the second term vanishes because the increased period 1 investment completely crowds out period 2 investment. Hence, in this case, the entire dynamic free-riding effect is due to the indirect *crowding-out* effect; that is,  $\frac{dV_2}{dI_1} \Big|_{I_1^L} = \frac{\delta}{n}$ . Substituting back into the first-order condition for the equilibrium period 1 proposal, 5, gives:

$$u'(I_1^L) + \frac{\delta}{n} = \frac{2}{n+1}. \quad (7)$$

Thus, the crowding-out effect actually *reduces* the free-rider problem, since the (interior) value of  $I_1^L$  that solves (7) is strictly higher than the solution if  $\delta = 0$  and is actually increasing in  $\delta$ . The intuition behind this is that the period 1 proposer can reduce the side payments to coalition members by increasing  $V_2$  (by freeing up more period 2 budget for private allocations) and raise her own payoff at the same time.

**Case 2: Corner solution,**  $I_2^L(I_1^L) = 0$ . If in equilibrium, the constraint  $0 \leq I_2$  binds, then  $\frac{dI_2}{dI_1} = 0$ , and the first term vanishes. In this case, investment in period 1 will not substitute for investment in period 2 at the margin. Hence, in this case, the entire dynamic free-riding effect is due to the direct *durability* effect. That is,  $\frac{dV_2}{dI_1} \Big|_{I_1^L} = \delta u'(\delta I_1)$ . Substituting back into the first-order condition for the equilibrium period 1 proposal, 5, gives:

$$u'(I_1^L) + \delta u'(\delta I_1^L) = \frac{2}{n+1}. \quad (8)$$

Given the functional form used in our experiments  $u(g) = Ag^\alpha$ , the equilibrium investment levels in the legislative bargaining game are given by:

$$\text{if } \delta^{1-\alpha} < 1 - \delta \frac{n+1}{2n} \text{ then } \begin{cases} I_1^{L^*} = \left[ \frac{1 - \frac{n+1}{2n} \delta}{\frac{n+1}{2} A \alpha} \right]^{\frac{1}{\alpha-1}} \\ I_2^{L^*} = \left[ \frac{1}{\frac{n+1}{2} A \alpha} \right]^{\frac{1}{\alpha-1}} - \delta \left[ \frac{1 - \frac{n+1}{2n} \delta}{\frac{n+1}{2} A \alpha} \right]^{\frac{1}{\alpha-1}} \end{cases}$$

$$\text{if } \delta^{1-\alpha} \geq 1 - \delta \frac{n+1}{2n} \text{ then } \begin{cases} I_1^{L^*} = \left[ \frac{1}{\frac{n+1}{2} A \alpha (1 + \delta^\alpha)} \right]^{\frac{1}{\alpha-1}} \\ I_2^{L^*} = 0 \end{cases}$$

Finally, we note that it is straightforward to generalize the analysis presented here to any quota voting rule, where a proposal passes if a winning coalition requires at least  $q$  individuals, where  $q$  is any integer from 1 to  $n$ . The main idea is that the free-riding problem is linked directly to the fact that a proposer will only internalize the value to  $q$  members of the legislature since that is all she needs for the proposal to pass. When  $q = n$ , there is no free-rider problem, and the optimal public investment is the equilibrium investment.