Supplementary Material for
“What Makes Voters Turn Out: The Effects of Polls and Beliefs”

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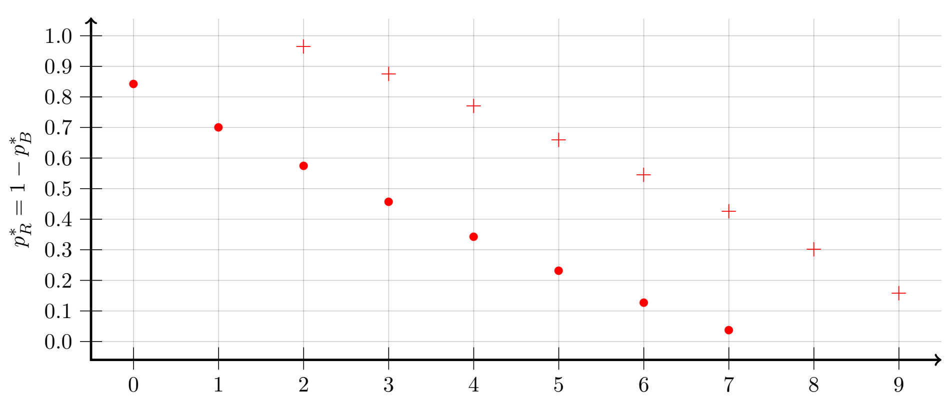
Abstract

We provide details of the theoretical analysis pertaining to our Lab Polls Treatment as well as the analysis of a model explaining our data with a preference for voting with the winner.

1 Endogenous Polls and Palfrey-Rosenthal Equilibria

Figure 1 below depicts the set of equilibria a-la Palfrey and Rosenthal (1983) for each number of individuals preferring the red jar. The figure depicts the probability that red supporters participate, p_R^*. The probability the blue supporters participate is given by $p_B^* = 1 - p_R^*$. If subjects were reporting truthfully in the polls, these would be the equilibria to expect at the voting stage. As can be seen, for intermediate number of red supporters, between 2 and 7, there are two possible equilibria. There are therefore $2^6 = 64$ possible equilibrium selections, mappings that determine the equilibrium played for any realization of red supporters.

Figure 1: Voting Propensities of Red Supporters according to Palfrey-Rosenthal Equilibria



Notes: The horizontal axe depicts the number of individuals preferring the red jar.

In order to see that the babbling equilibrium is the only symmetric equilibrium in which subjects use pure strategies at the communication stage, we do the following. First, for each of the 64 selections, we compare the expected utility of a subject who reports truthfully and then follows the selected equilibrium strategy to both 1. the expected utility of a subject who lies in the poll (stating she will vote for her less preferred alternative) and then follows the resulting equilibrium strategy and 2. the expected utility of a subject who lies in the poll and then abstains. In other words, we consider two types of deviations, one entailing lying in the poll and following the equilibrium prescription that ensues, and one entailing lying in the poll and abstaining. For 63 out of the possible 64 selections, one of these deviations is beneficial. For instance, suppose we consider the selection under which the equilibria denoted by "+" in Figure R1_1 are selected whenever possible, the expected utility from truthful behavior is 163.8, while the expected utility from lying in the poll and abstaining is 179.8 (which is greater than the expected utility from lying in the poll and voting). If we consider the selection under which the equilibria denoted by "·" in Figure R1_1 are selected whenever possible, the expected utility from truthful behavior is 29.6, while the expected utility from lying in the poll and voting is 66.3 (which is greater than the expected utility from lying in the poll and abstaining). For the one selection for which neither of these deviations provide an improvement, there is a different deviation, one entailing lying in the poll followed by abstention for low red reports, and participation for high red reports in the poll.

2 Voting with the Winner

In this section we consider a model that incorporates the Callander preferences (Callander, 2007, 2008) in which voters receive an additional benefit from voting for the winner of an election as specified in Section of 4.4 of the paper. We start by considering a specification with a homogeneous population characterized by one positive parameter a that captures the utility impact of voting for the winner. This model performs fairly poorly in terms of matching our documented comparative statics regarding voting propensities. This is not very surprising given the large variation in individually optimal parameters a estimated across subjects and reported in Figure 3 in the paper. We then proceed to analyze equilibria of the voting model in which there are two types of voters: those with a positive value of a and those who do not value voting for the winner per-se. It turns out that allowing for only two types of voters explains much of our data, while allowing for more types has a relatively small additional benefit.

2.1 Callander Equilibrium with One Type

Consider the model in which all voters derive additional benefit from voting with the winner, captured by the parameter a, as in the body of the paper. Suppose first that all voters have the same parameter $a > 0$.

Given our experimental data, the best fitting parameter a in this case is $a = 25$. In our No Polls treatment, there exists then a unique equilibrium in each of the two cost treatments. In the low cost treatment ($c = 25$), in the unique equilibrium, all voters vote with probability one. In the high cost treatment ($c = 50$), in the unique equilibrium, all voters vote with probability $\gamma = 0.443$. When information is introduced in the form of Perfect Polls, multiple equilibria arise. Specifically, in the low cost treatment, there are three possible equilibria: one in which all majority voters turn out, while all minority voters abstain, another in which all minority voters turn out, while all majority voters abstain, and, finally, a third equilibrium in which majority voters vote with probability $\gamma^{\text{maj}} = 0.016$, while minority voters vote with probability one. When costs of voting are high, there exists a unique equilibrium in which majority voters vote with probability $\gamma^{\text{maj}} = 0.470$, while minority voters vote with probability $\gamma^{\text{min}} = 0.940$.[1]

While the Perfect Polls treatment might feature multiple equilibria (as discussed above), equilibria in which both minority and majority voters mix between voting and abstaining would necessarily entail minority voters to vote with *higher* probability than majority voters. The intuition for this hinges on the cost-benefit analysis of participation. Indeed, while the costs of voting are the same for members of the two groups, the benefit is different and depends on the probability of being pivotal and on the likelihood of collecting payoffs associated with voting with the winner. Ceteris paribus, the introduction of a benefit from voting for the winner increases participation of the majority group, which reduces both the probability of being pivotal for the minority member (compared with the standard pivotal voter model) as well as the probability of collecting the benefit from voting with the winner. Therefore, to make up for these two effects and achieve equality between expected benefit and expected costs, members of the minority group should vote more than members of the majority group.

The intuition presented above suggests that unless one focuses on the extreme equilibria configurations, in which either minority voters or majority voters always abstain, the Callander model with one type produces the same comparative statics predictions regarding turnout as the standard pivotal voter model, which is clearly rejected by our data. Indeed, as we discuss in the main text of the paper, in all our treatments, we observe that members of the majority group participate at significantly higher rates than members of the minority group.

As we describe in the paper, there is a non-trivial distribution of individually estimated parameters a. The observations here illustrate that this heterogeneity may play an important role. Indeed, the Callander model with one type cannot accommodate the patterns observed in our experimental data.

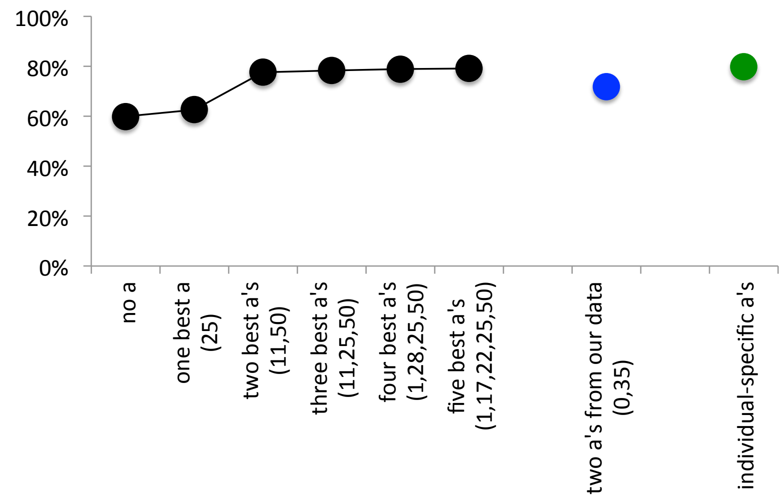
2.2 Number of Types

Given the poor performance of the Callander model with one type, we consider the Callander model with more than one type of voter. To motivate the focus on a model with two types, we first inspect the number of types that are important to consider to explain behavior in our data. To answer this, we report in Figure 2 the fraction of choices consistent with the model with different numbers of types.

As evident from Figure 2, there is a significant improvement in the fit of the model as we go from one type to

[1] These values are derived numerically.

Figure 2: Fraction of data consistent with Callander model with different number of types



Notes: Here we pool together data from all three informational treatments (No Polls, Perfect Polls and Lab Polls) as well as from two different cost treatments. For each category we determine the best values of a's that fit our data and then compute the fraction of choices that are consistent with model with this number of types. The last data point corresponds to the score obtained when using individual specific values of a.

two types, while further increase in the number of types produces only a modest benefit. Indeed, the model with two best-fitting types explain 78% of our data, while the best fit of the model obtained with individual specific values of a achieves the fit of 80%.

2.3 Callander Equilibria with Two Types

In the remainder of this section, we focus on the model with two types of subjects: one type derives no extra benefit from voting with the winner ($a = 0$) and the other type is characterized by a strictly positive value of a. Based on the individual specific estimation of a values presented in Figure 3 in the main text of the paper, the average value of a for subjects with strictly positive a is $a = 35$.[2] The model with these two types of a ($a = 0$ and $a = 35$) performs quite well in terms of aggregate scores: it explains 72% of all voting decisions.

In the model we analyze below, we assume that each agent is of type 0 or $a > 0$ with $\Pr(a) = q, q \in (0, 1)$. Types are i.i.d. and private information. There are two states (namely, the red and blue jars) and we assume that an individual's private preference coincides with the state (namely, red preferred when the state is red, and blue preferred when the state is blue) with probability p (in our experiments, $p = 2/3$). Preference types are independent of the preferred alternative.

[2]Specifically, we count subjects with $a < 2$ as the type with $a = 0$ to allow for some error in estimation. Therefore, the average value of a conditional on a being strictly greater than 2 is 35.

2.3.1 No Polls

Following the analysis in the main text of the paper, we will focus on quasi-symmetric equilibria in which type-0 vote with probability γ_0 and type-a vote with probability γ_a.

Denote by $P_{piv}(k)$ the probability that an agent is pivotal when k other agents participate. As before, $P_{piv}(0) = 1$, $P_{piv}(1) = 1/2$, and for any $j = 1, ..., \lfloor (n-1)/2 \rfloor$,

$$P_{piv}(2j) = \binom{2j}{j} p^j (1-p)^j \text{ and } P_{piv}(2j+1) = \binom{2j+2}{j+1} p^{j+1} (1-p)^{j+1}.$$

Denote by $P_{win}(k)$ the probability that the agent votes for the winner when k other agents participate. $P_{win}(0) = 1$, $P_{win}(1) = 1/2$, and for any $j = 1, ..., (n-1)/2$,

$$\begin{aligned} P_{win}(2j) &= {}^{2j}_{m=j} \binom{2j}{m} \left[\underset{\substack{\text{(state corresponds to} \\ \text{preferred alternative)}}}{p} \cdot p^m (1-p)^{2j-m} + \underset{\substack{\text{(state corresponds to} \\ \text{non-preferred alternative)}}}{(1-p)} \cdot p^{2j-m} (1-p)^m \right] = \\ & {}^{2j}_{m=j} \binom{2j}{m} \left[p^{m+1} (1-p)^{2j-m} + p^{2j-m} (1-p)^{m+1} \right] \end{aligned}$$

$$P_{win}(2j+1) = \frac{1}{2} P_{piv}(2j+1) + {}^{2j+1}_{m=j+1} \binom{2j+1}{m} \left[p^{m+1} (1-p)^{2j+1-m} + p^{2j+1-m} (1-p)^{m+1} \right]$$

Finally, denote by $\gamma^* \equiv (1-q)\gamma_0 + q\gamma_a$ the expected voting propensity of a randomly drawn voter.

If $\gamma_0 \in (0,1)$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} P_{piv}(k) = c.$$

If 0-types use a corner strategy, say $\gamma_0 = 0$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} P_{piv}(k) \leq c.$$

Similarly, if $\gamma_0 = 1$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} P_{piv}(k) \geq c.$$

For the a-types, if $\gamma_a \in (0,1)$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} [P_{piv}(k) + aP_{win}(k)] = c.$$

As before, if a-types use a corner strategy, say $\gamma_a = 0$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} [P_{piv}(k) + aP_{win}(k)] \leq c.$$

Last, if $\gamma_a = 1$,

$$\frac{V}{2} * \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} [P_{piv}(k) + aP_{win}(k)] \geq c.$$

In equilibrium, voters of each type best respond given the behavior of others. For the parameters used in our experiments, there exists a unique equilibrium in the No Polls treatment for both low and high costs of voting. Specifically, we compute equilibria for $p = 2/3$, $a = 35$, and $q = 0.58$, which is the fraction of subjects characterized by a strictly positive value of a (in our classification here, those individuals with an estimated $a \geq 2$). In the low cost treatment, in the unique equilibrium, a-type subjects vote with probability one ($\gamma_a = 1$), while 0-type subjects mix and vote with probability $\gamma_0 = 0.07$. In the high cost treatment, in the unique equilibrium, a-type subjects vote with probability one ($\gamma_a = 1$), while 0-type subjects abstain ($\gamma_0 = 0$).[3]

2.3.2 Perfect Polls

The quasi-symmetric equilibria in the Perfect Polls treatment consists of voting propensities for each type of voter in each of the two groups of voters, expected majority and expected minority. Specifically, the equilibrium is characterized by four values $(\gamma_0^{\text{maj}}, \gamma_0^{\text{min}}, \gamma_a^{\text{maj}}, \gamma_a^{\text{min}})$, where $\gamma_x^{\text{maj}}, \gamma_x^{\text{min}}$ denote the turnout probability of an x-type in the majority or minority, respectively, where $x \in \{0, a\}$. The model in this case does not produce a unique equilibrium prediction. The full characterization of the set of equilibria is beyond the scope of the current paper. In what follows, we focus on one specific equilibrium configuration, which has the potential to fit the observed empirical patterns documented in our experimental data. In this configuration, a-type voters that belong to the expected majority always vote, i.e., $\gamma_a^{\text{maj}} = 1$, voters with 0-type from either group abstain, i.e., $\gamma_0^{\text{maj}} = \gamma_a^{\text{min}} = 0$, and a-type voters from the expected minority group mix between voting and abstaining, i.e., $\gamma_a^{\text{min}} \in (0,1)$. We now illustrate that we can find $\gamma \equiv \gamma_a^{\text{min}} \in (0,1)$ consistent with equilibrium.

Suppose the number of voters is odd, $n = 2m+1$, where $m \in \mathbb{N}$, as in our experiments. In line with our

[3]Again, these values are obtained from numerically searching for equilibria.

previous notation, we denote by P_{piv}^{min} (or P_{piv}^{maj}) the probability of an agent being pivotal when in the likely minority (or majority) The constellation above is an equilibrium if the following four conditions are satisfied:

1. a-type voters from the minority group are indifferent between voting and not voting, i.e.,

$$\frac{V}{2}P_{piv}^{\text{min}} + aP_{win}^{\text{min}} = c, \tag{1}$$

   where

$$\begin{aligned} P_{piv}^{\text{min}} &= \sum_{k=0}^{m}\binom{n-1}{2k}\binom{2k}{k}(pq)^k\left((1-p)q\gamma\right)^k\left(1-pq-(1-p)q\gamma\right)^{n-2k-1} + \\ &\quad \sum_{k=0}^{m-1}\binom{n-1}{2k+1}\binom{2k+1}{k}(pq)^{k+1}\left((1-p)q\gamma\right)^k\left(1-pq-(1-p)q\gamma\right)^{n-2k-2}. \end{aligned}$$

   and

$$P_{win}^{\text{min}} = \frac{1}{2}P_{piv}^{\text{min}} + \sum_{k=0}^{m-1}\sum_{j=k+1}^{n-1-k}\binom{n-1}{j+k}\binom{j+k}{j}(pq)^k\left((1-p)q\gamma\right)^j\left(1-pq-(1-p)q\gamma\right)^{n-j-k-1}.$$

2. 0-type voters from the minority group prefer to abstain, i.e.,

$$\frac{V}{2}P_{piv}^{\text{min}} \leq c. \tag{2}$$

3. a-type voters from majority group prefer to vote, i.e.,

$$\frac{V}{2}P_{piv}^{\text{maj}} + aP_{win}^{\text{maj}} \geq c, \tag{3}$$

   where

$$\begin{aligned} P_{piv}^{\text{maj}} &= \sum_{k=0}^{m}\binom{n-1}{2k}\binom{2k}{k}(pq)^k\left((1-p)q\gamma\right)^k\left(1-pq-(1-p)q\gamma\right)^{n-2k-1} + \\ &\quad \sum_{k=0}^{m-1}\binom{n-1}{2k+1}\binom{2k+1}{k}(pq)^{k}\left((1-p)q\gamma\right)^{k+1}\left(1-pq-(1-p)q\gamma\right)^{n-2k-2} \end{aligned}$$

   and

$$P_{win}^{\text{maj}} = \frac{1}{2}P_{piv}^{\text{maj}} + \sum_{k=0}^{m-1}\sum_{j=k+1}^{n-1-k}\binom{n-1}{j+k}\binom{j+k}{j}(pq)^j\left((1-p)q\gamma\right)^k\left(1-pq-(1-p)q\gamma\right)^{n-j-k-1}.$$

4. 0-type voters from the majority group prefer to abstain, i.e.,

$$\frac{V}{2}P_{piv}^{\text{maj}} \leq c \tag{4}$$

Given our parameters, there is a unique equilibrium of this form in each of our two costs treatments. Specifically, a-type voters that belong to the expected minority vote with probability $\gamma_a^{\text{min}} = 0.40$ in the low-cost treatment, and with a higher probability $\gamma_a^{\text{maj}} = 0.90$ in the high-cost treatment.

2.3.3 Fit of Callander Model with Two Types

We now compare the expected turnout in the equilibria identified above with the observed turnout in our experiments. As evident from Table 1 here, the Callander model with two types of voters generates predictions consistent with several patterns in our data. In particular, comparative statics with respect to cost track those observed in the data. More importantly, in the Perfect Polls treatment, for both costs, individuals who are likely in the majority vote with higher propensity than those in the likely minority.

Table 1: Expected Turnout in the Callander model with Two Types and Observed Turnout

	No Polls		**Perfect Polls**			
	c=25	c=50	c=25		c=50	
			Majority	Minority	Majority	Minority
Expected Turnout (Theory)	0.61	0.58	0.58	0.23	0.58	0.52
Observed Turnout (Data)	0.55	0.43	0.63	0.38	0.52	0.27
	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)

Notes: Robust standard errors are clustered at the group level and reported in the parenthesis.